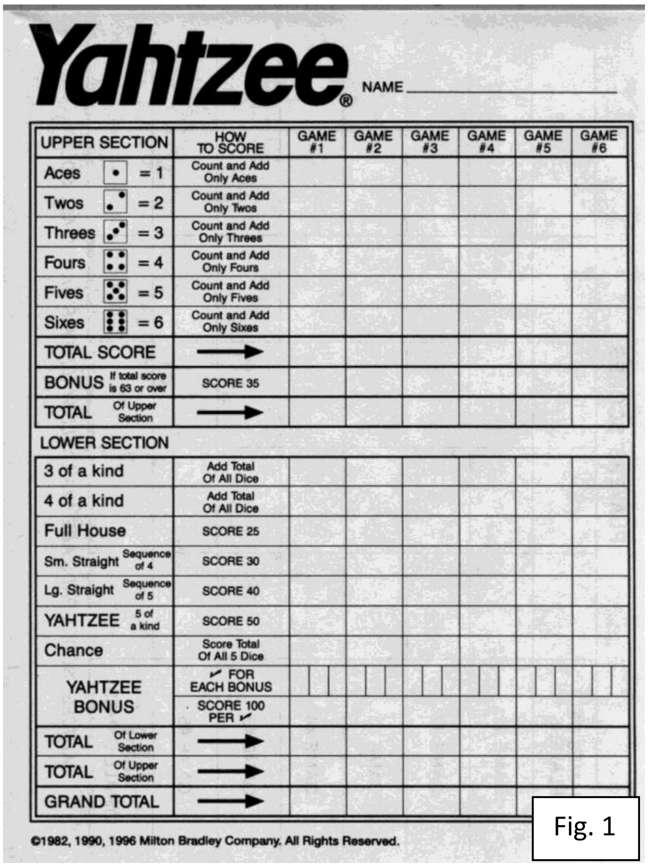
Investigating the Probabilities of getting a Yahtzee when varying the number of dice.

Worsham, Sally. "How to Play: Yahtzee." ParentDish. April 20, 2010. www.parentdish.com/2010/04/20/yahtzee-game/ Accessed March 13, 2015

1.0 Introduction

Yahtzee is a dice game invented by Milton Bradley and now owned by Hasbro, an American multinational toy and board game company[1]. The goal of the game is to score the most points by fulfilling certain scoring combinations. There exist 13 different scoring combinations. More so scoring combinations have varying point values, some which are fixed and others which have a cumulative value of the dice[2]. You play the game with 5 dice which can be rolled up to 3 times in a turn. Next to the Large Straight (Five sequential dice, 40 points), Small Straight (Four sequential dice, 30 points), Full House (A Three-Of-A-Kind and a Pair, 25 points), Four-Of-A-Kind (four dice showing the same number, Sum of all dice; highest possible score 29) and Three-Of-A-Kind (three dice showing the same number, sum of all dice). A Yahtzee is the highest scoring combination with 50 points. You score a Yahtzee when all of the five die faces are the same. Fig. 1[3] is showing the typical scoring card of Yahtzee on which all scoring combinations are shown.


Yahtzee® NAME

UPPER SECTION	HOW TO SCORE	GAME #1	GAME #2	GAME #3	GAME #4	GAME #5	GAME #6
Aces = 1	Count and Add Only Aces						
Twos = 2	Count and Add Only Twos						
Threes = 3	Count and Add Only Threes						
Fours = 4	Count and Add Only Fours						
Fives = 5	Count and Add Only Fives						
Sixes = 6	Count and Add Only Sixes						
TOTAL SCORE	→						
BONUS If total score is 63 or over	SCORE 35						
TOTAL Of Upper Section	→						
LOWER SECTION							
3 of a kind	Add Total Of All Dice						
4 of a kind	Add Total Of All Dice						
Full House	SCORE 25						
Sm. Straight Sequence of 4	SCORE 30						
Lg. Straight Sequence of 5	SCORE 40						
YAHTZEE 5 of a kind	SCORE 50						
Chance	Score Total Of All 5 Dice						
YAHTZEE BONUS	✓ FOR EACH BONUS						
	SCORE 100 PER ✓						
TOTAL Of Lower Section	→						
TOTAL Of Upper Section	→						
GRAND TOTAL	→						

Fig. 1

1.1

When I was younger I used to play Yahtzee with my whole family on regular bases. My parents always told me that it is luck to win and get the highest score. As a child I believed what they were saying. But by observing recently played Yahtzee games, it was noticeable that whoever scores a Yahtzee in a game is likely to be the winner at the end. When we did probability calculations in our Mathematics Standard Level class about other dice games, I realized that there must be a link to my Internal Assessment topic of how to calculate the probability of getting a Yahtzee. So my aim in this investigation is to find the probability of scoring a Yahtzee in three rolls.

[1] "Yahtzee." Wikipedia. www.en.wikipedia.org/wiki/Yahtzee Accessed March 13, 2015
[2] "Yahtzee." Wikipedia. www.en.wikipedia.org/wiki/Yahtzee Accessed March 13, 2015
[3] "Yahtzee." Yahtzee. www.math.cornell.edu/~mec/2006-2007/Probability/Yahtzee.htm Accessed March 13, 2015

2.0 Investigation – First Attempt

Before I could calculate all the probabilities of scoring a Yahtzee after 3 or less rolls I needed to figure out the probabilities of every single possibility the dice could be rolled. A normal game of Yahtzee is played with 5 dice so you could score a Yahtzee, a 4 of a kind, a 3 of a kind, a 2 of a kind and a 1 of a kind on your first roll. But when all dice show a different number you probably would throw all of them again to have a higher chance of possibly scoring a Yahtzee.

For scoring a Yahtzee all dice have to show the same number. So if I want to score a Yahtzee of specific number, let's take 3's, then the probability that one die shows a 3 is $\frac{1}{6}$ and the probability that it doesn't is $\frac{5}{6}$. If I want that all 5 dice show a 3 than I have to multiply the probabilities of all single dice together:

$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{6^5}=\frac{1}{7776}\approx 0.01\%$$

But as there are are 6 different numbers on a die I had to multiply this value with 6, so I get the general probability of scoring a Yahtzee on the first roll.

$$6\times\frac{1}{6^5}=\frac{6}{6^5}=\frac{1}{1296}\approx 0.08\%$$

Scoring a Yahtzee on the first roll is not that likely to happen as the probability is relatively small. So other scoring combinations could be rolled such as 4 of a kind, 3 of a kind and so on. I tried to calculate the probabilities of these possibilities in the tree diagram in Fig. 2. Throwing a 4 of a kind means that 4 dice show the same number and one die shows a different one. This means that the calculation has to be:

$$6\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}=\frac{5}{6^4}=\frac{5}{1296}\approx 0.39\%$$

If have to multiply it by 6 again as there are still 6 different numbers on a die. As only one die is left after getting a 4 of a kind, the probability of scoring a Yahtzee in the second roll is equal to 0.064% ($\frac{5}{1296}\times\frac{1}{6}=\frac{5}{6^5}=\frac{5}{7776}\approx 0.064\%$) and after the third roll it is equal to 0.054% ($\frac{5}{1296}\times\frac{5}{6}\times\frac{1}{6}=\frac{5^2}{6^6}=\frac{25}{46656}\approx 0.054\%$)

Throwing a 3 of a kind means that 3 dice show the same number and the 2 other ones show a different one.

$$6\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}=\frac{5^2}{6^4}=\frac{25}{1296}\approx 1.93\%$$

For the second roll there are now more different possibilities that could happen for example I could score a Yahtzee, so the 2 left dice show the same number as the 3 of a kind from the first roll. I could score a 4 of a kind which means one of the two left dice shows the same number as the 3 of a kind from the first roll or I could still have a 3 of a kind as none of the two left dice is showing the same number as the 3 of a kind from the first roll. These possibilities and probabilities are shown in Fig. 2. And so are the probabilities of scoring a Yahtzee after the third roll.

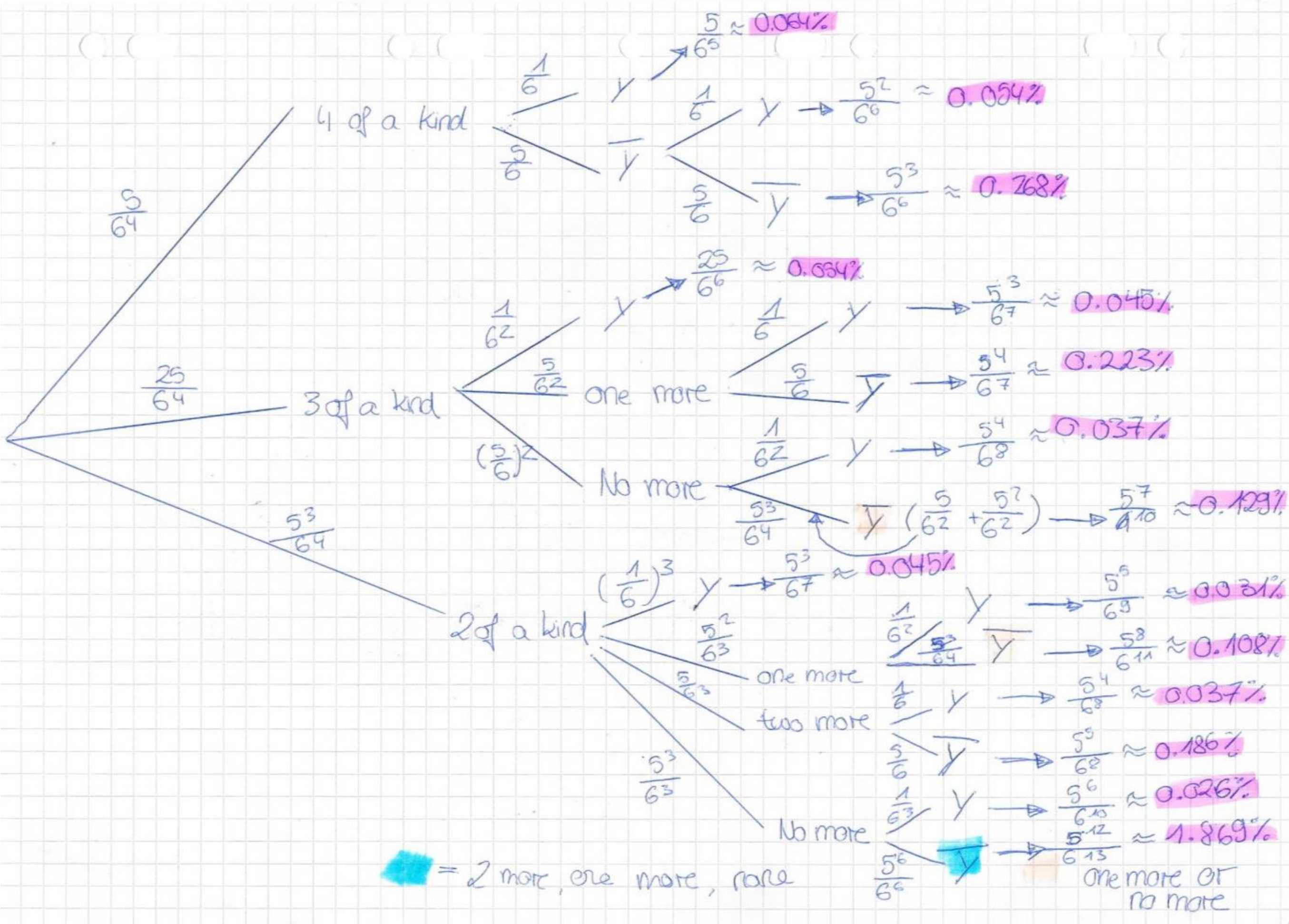
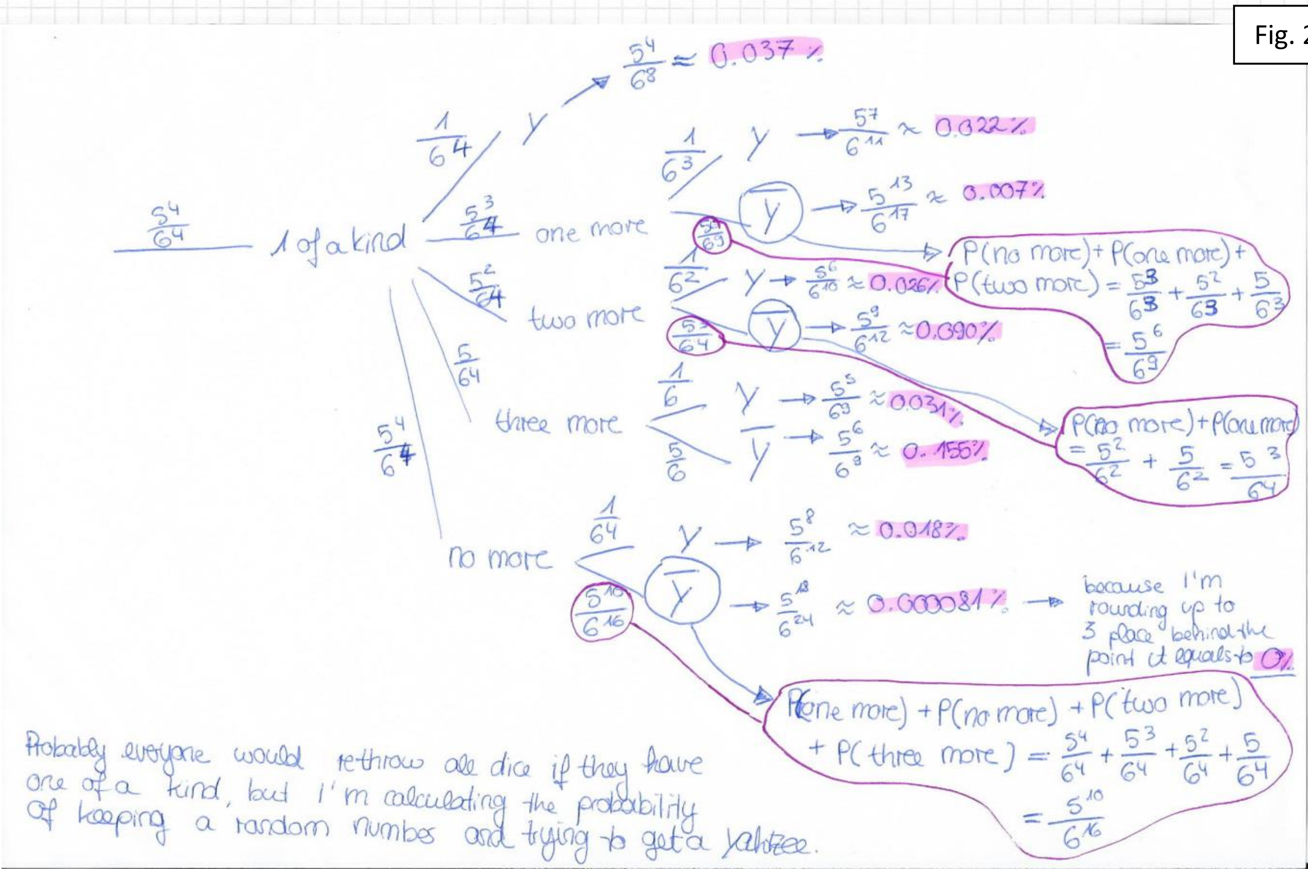

4 of a kind
3 of a kind
2 of a kind
one more
No more
two more
= 2 more, one more, none
one more or no more

Fig. 2


1 of a kind
one more
two more
three more
no more
P(no more) + P(one more) + P(two more)
P(no more) + P(one more)
P(one more) + P(no more) + P(two more) + P(three more)
because I'm rounding up to 3 place behind the point it equals to 0%
Probably everyone would rethrow all dice if they have one of a kind, but I'm calculating the probability of keeping a random numbers and trying to get a Yahtzee.

I soon realized that something wasn't right with my calculations though the first calculation of scoring a Yahtzee with 5 dice in the first roll seems right. I realized that there are different combinations possible of scoring a 4 of a kind, a 3 of a kind and a 2 of a kind. The order in which the dice were arranged after the roll was random (AAABC, ABACA, AABCA... etc.). To get on the right path with my calculations I decided to calculate the probabilities of scoring a Yahtzee with only 2 dice in the beginning as this would be easier and clearer. After achieving these calculations I decided to increase the number of dice, so that at the end I could calculate the probabilities scoring a Yahtzee with 5 dice easier as some of the calculations are the same throughout the process.

2.1 Probability of getting a Yahtzee with 2 dice

A Yahtzee with 2 dice means rolling a double in 3 throws. In Fig. 3 you can see the tree diagram of the probabilities of all possibilities that you can get with 2 dice in 3 rolls. I renamed 1 of a kind to 0 of a kind as everyone would throw all dice again if none of the dice show the same number. There are only two different combinations possible. The 2 dice could show the same number (Yahtzee) or two different numbers (0 of a kind).

To score a Yahtzee in the first roll the calculation is going to be:

$$6 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6} \approx 16.7\%$$

And to score a 0 of a kind the calculation would be:

$$6 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{6} \approx 83.3\%$$

If you have scored a 0 of a kind in your first roll the probability of scoring a Yahtzee in the second roll would be same as in the first roll ($\frac{1}{6}$). And if you have scored a 0 of a kind in the second roll, the probabilities are the same in the third roll.

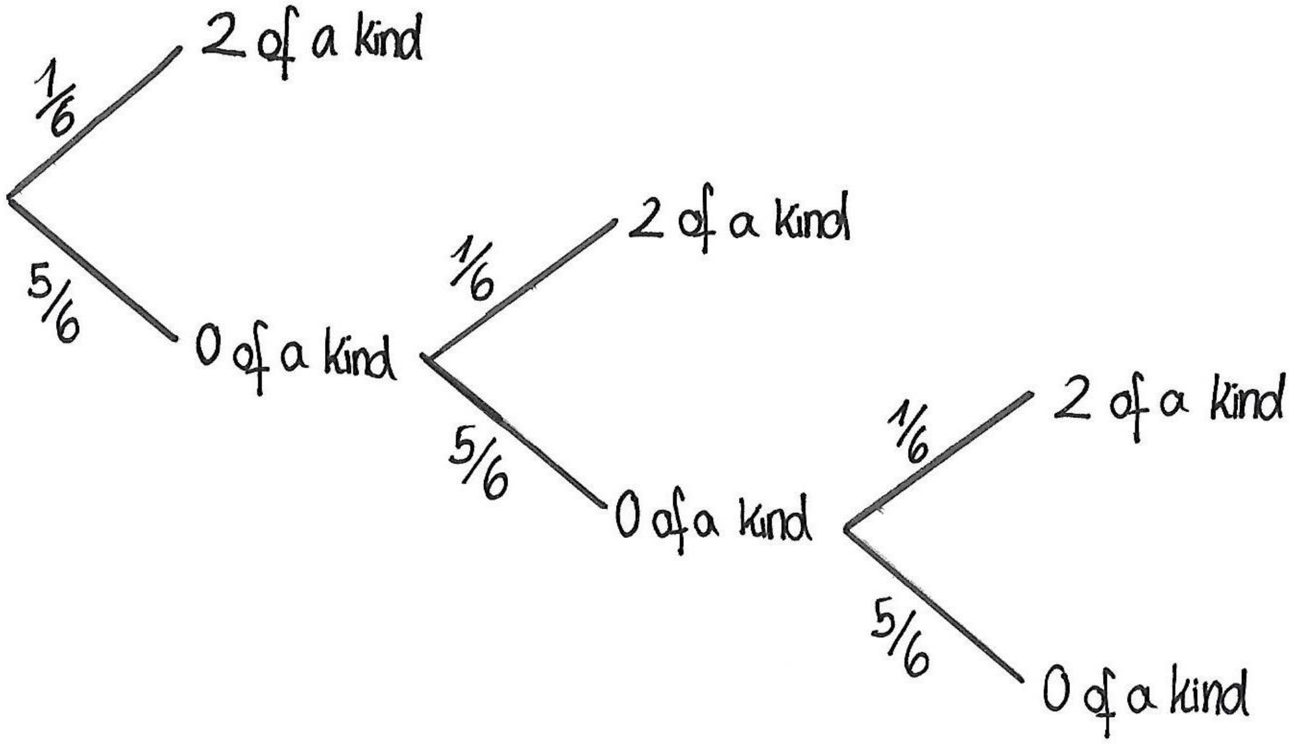


Fig. 3

There are 3 different possibilities of scoring a Yahtzee with 2 dice in 3 rolls. So to calculate the overall probability of scoring a Yahtzee I need to add all possibilities together.

$$\frac{1}{6} + \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) = \frac{1}{6} + \frac{5}{6^2} + \frac{5^2}{6^3} = \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{36}{216} + \frac{30}{216} + \frac{25}{216} = \frac{91}{216} \approx 42.1\%$$

If you play the game of Yahtzee with only 2 dice than the probability would be 42.1% that you would score a Yahtzee which means you would approximately score a Yahtzee every third time ($\frac{100}{42.1} \approx 2.37$).

2.2 Probability of getting a Yahtzee with 3 dice

A Yathzee with 3 dice means rolling a triple in 3 throws. In Fig. 4 you can see the tree diagram of the probabilities of scoring a Yahtzee with 3 dice in 3 rolls. As in the game with 2 dice here again there is no 1 of a kind but a 0 of a kind. With 3 dice the number of possible combinations have increased. You could have a Yahtzee (in the tree diagram: 3 of a kind), a 2 of a kind and a 0 of a kind in your first roll.

To score a Yahtzee in the first roll the calculation is going to be:

$$6 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6^2} = \frac{1}{36} \approx 2.78\%$$

To score a 2 of a kind the calculation must be:

$$6 \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{6^2} = \frac{5}{36}$$

But as there are 3 different combinations possible (AAB, ABA, BAA) in which order the dice are rolled it means I have to multiply the answer by 3.

So the right equationfor rolling a 2 of a kind is:

$$6 \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times 3 = \frac{15}{36} \approx 41.67\%$$

When you have a 2 of a kind in your first roll only one die is left. You could whether score a Yahtzee on your second goal so you have to multiply $\frac{15}{36} \times \frac{1}{6} = \frac{15}{216} \approx 6.94\%$ or you could still have only 2 of a kind. When you still only have 2 of a kind you could score a Yahtzee in your third roll (probabilities in Fig. 4)

But you could also roll 3 different number, a 0 of a kind, in your first roll (ABC). The equation to calculate this probability would be:

$$6 \times \frac{1}{6} \times \frac{5}{6} \times \frac{4}{6} = \frac{20}{36} \approx 55.56\%$$

You have to multiply the probability of getting one number ($\frac{1}{6}$) with the probability of getting a different number ($\frac{5}{6}$) and the probability of getting again a different number ($\frac{4}{6}$). If you would multiply twice by $\frac{5}{6}$ you risk of calculating the probability of a pair so you have to multiply it by $\frac{4}{6}$. In your second roll you could roll a Yahtzee, a 2 of a kind or again a 0 of a kind. The probabilities are the same as in your first roll (For the probabilities and possible combinations in your third roll look at Fig. 4, p. 9)

There are 6 possible ways of getting a Yahtzee when you play the game with 3 dice. To calculate the overall probability of getting a Yahtzee with 3 dice I need to add all probabilities of each single wlay together.

$$\frac{1}{6^2} + \frac{15}{6^3} + \frac{75}{6^4} + \frac{20}{6^4} + \frac{300}{6^5} + \frac{400}{6^6} = \frac{1296}{46656} + \frac{3240}{46656} + \frac{2700}{46656} + \frac{720}{46656} + \frac{1800}{46656} + \frac{400}{46656}$$
$$= \frac{10156}{46656} \approx 21.8\%$$

If you would play the game of Yahtzee with only 3 dice the probability that you will score a Yahtzee in 3 rolls is 21.8%. This means a player would score a Yahtzee approximately every fifth time ($\frac{100}{21.8} \approx 4.59$). So the probability of scoring a Yahtzee with 3 dice (21.8%) is 20.3 % less than the probability of scoring a Yahtzee when playing with 2 dice (42.1%). The chance to score a Yahtzee with 2 dice is therefore higher than scoring a Yahtzee with 3 dice.

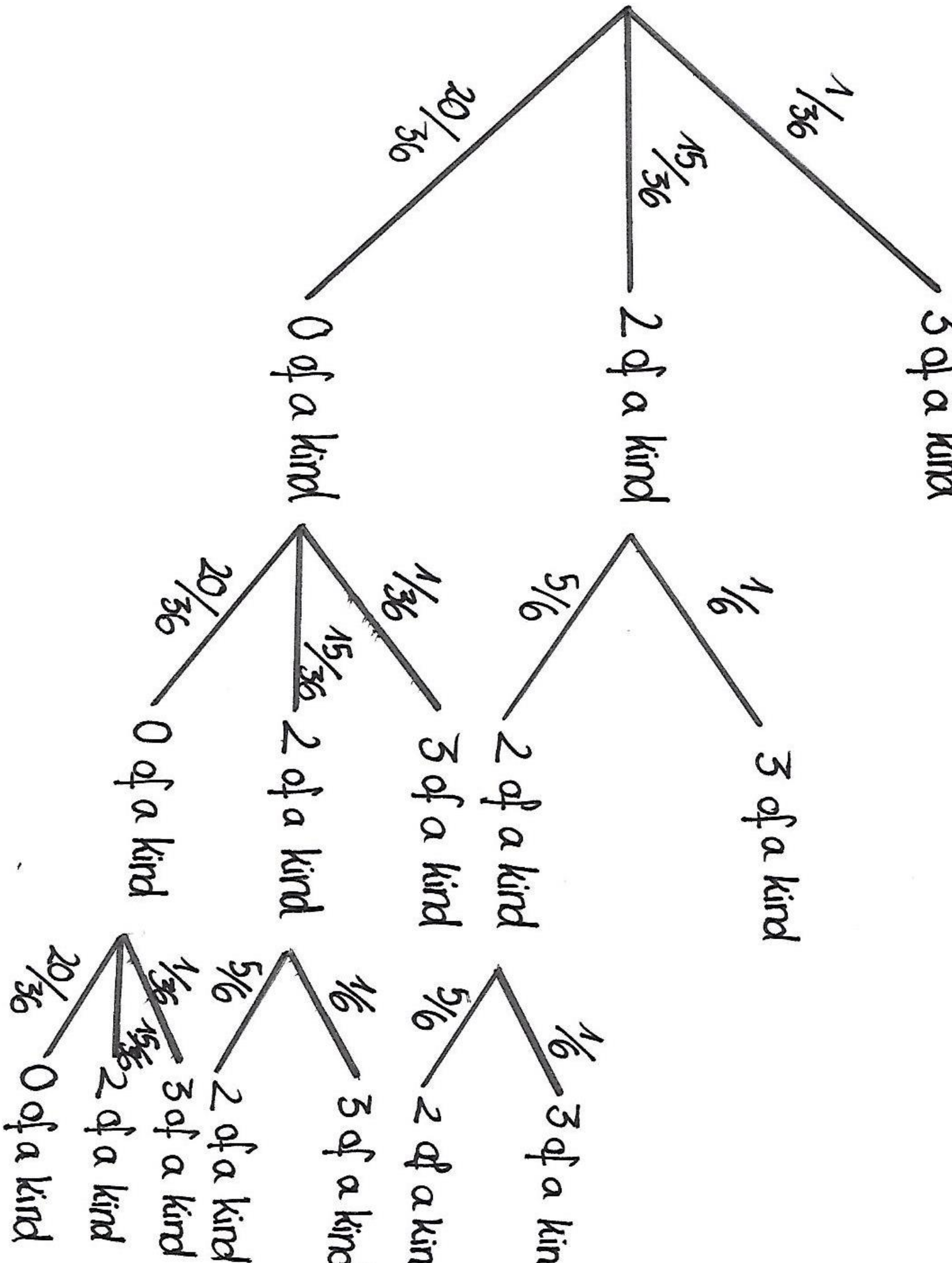


Fig. 4

2.3 Probability of getting a Yahtzee with 4 dice

Scoring a Yahtzee with 4 dice means that all 4 dice are showing the same faces. In Fig. 7 (p. 12) you can see the tree diagram of all possible combination that could be scored if you would play the game of Yahtzee with only 4 dice. This time there will be even a higher number of ways in which you can score a Yahtzee. In your first roll you could have a Yahtzee (in the tree diagram a 4 of a kind), a3 of a kind, a 2 of a kind and a 0 of a kind.

The equation for scoring a Yahtzee in your first roll would be:

$$6 \times \left(\frac{1}{6}\right)^4 = \left(\frac{1}{6}\right)^3 = \frac{1}{216} \approx 0.46\%$$

The equation for getting a 3 of a kind in your first roll has to be:

$$6 \times \frac{5}{6^4} \times 4 = \frac{5}{6^3} \times 4 = \frac{20}{216} \approx 9.26\%$$

You have to multiply it by 4 as there are 4 different possible combinations in which the dice could be arranged (AAAB, AABA, ABAA, BAAA). As only 1 die is left the probability of getting the same number is $\frac{1}{6}$ and the probability of getting a different number is $\frac{5}{6}$. So in your second roll you could rather get a Yahtzee or still have a 3 of a kind. If you score a 3 of a kind then you still have the chance to score a Yahtzee in the third roll.

As it turned out by trying to find the equation of getting a 2 of a kind in the first roll when using 4 dice it was too hard to find and I couldn't figure out how to get this equation. By creating a list of combinations on Excel I was able to count the number of possible combinations for scoring a 2 of a kind in the first roll. You can see a little section of the Excel table in Fig. 5.

Scoring possibilities when playing with 4 dice				**Numbers the dice could possibly show**						**of a kind**
				6	**5**	**4**	**3**	**2**	**1**	
1	1	1	1	0	0	0	0	0	4	4
1	1	1	2	0	0	0	0	1	3	3
1	1	1	3	0	0	0	1	0	3	3
1	1	1	4	0	0	1	0	0	3	3
1	1	1	5	0	1	0	0	0	3	3
1	1	1	6	1	0	0	0	0	3	3
1	1	2	1	0	0	0	0	1	3	3
1	1	2	2	0	0	0	0	2	2	2
1	1	2	3	0	0	0	1	1	2	2
1	1	2	4	0	0	1	0	1	2	2

Fig. 5

All in all there are 1296 different combinations that you could get on your first roll. Out of these 1296 possible combinations, 810 combinations were scoring a 2 of a kind in the first roll like shown in the table in Fig. 6.

Scoring Possiblities 4 Dice	
Of A Kind	**Number of Possible Combinations**
4	6
3	120
2	810
1	360

Fig. 6

So scoring a 2 of a kind in the first roll when playing with 4 dice must be:

$$\frac{810}{1296} = \frac{135}{216} \approx 62.50\%$$

When you get a 2 of a kind in your first roll you could get a Yahtzee (your other 2 dice show the same face as the pair in the first roll), a 3 of a kind (one of the two left dice is showing the face as the pair in the first roll) or still a 2 of a kind (none of the 2 dice is showing the same face as the pair in the first roll) in your second roll. For getting a 3 of a kind there are again 2 possible combination in which the dice could be arranged as only 2 dice are left (AB or BA). To get a 3 of a kind in your second roll supposed you have had a 2 of a kind in your first roll the equation is going to be:

$$\frac{1}{6} \times \frac{5}{6} \times 2 = \frac{10}{36}$$

In your third roll there are 2 possible ways in which you can score a Yahtzee, supposing you have score a 3 of a kind or a 2 of a kind in your second roll and a 2 of a kind in your first roll. The probabilities for getting a Yahtzee, a 3 of a kind or a 2 of a kind when you had a 2 of a kind in your second roll, would be the same as all the probabilities of the second roll after supposed you have a 2 of a kind in your first roll.

You possibly could get differnet numbers on every die (0 of a kind) in your first roll. The equation for this would be:

$$6 \times \frac{1}{6} \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} = \frac{60}{216} \approx 27.78\%$$

You have to multiply it by $\frac{4}{6}$ and $\frac{3}{6}$ as the last two dice can't have the same number as the second die. If the second die doesn't show the same number as the first die and the third die isn't allowed to show the same number as the first and the second die there are only 4 numbers which the third die could show. The same process is applied to the last die. For the second roll the probabilties of the different scoring combinations are the same as in the first roll. And so the probabilities of the different scoring combinations in the thrid roll are the same as in the second roll of the whole tree diagram.

There are 10 different ways of scoring a Yahtzee in 3 rolls when you play the game with only 4 dice. To calculate the overall probability of scoring a Yahtzee I need to add all the probabilities of he 10 different ways together.

$$\frac{1}{6^3}+\frac{20}{6^4}+\frac{100}{6^5}+\frac{135}{6^5}+\frac{1350}{6^6}+\frac{3375}{6^7}+\frac{60}{6^6}+\frac{1200}{6^7}+\frac{8100}{6^8}+\frac{3600}{6^9}$$
$$=\frac{46656}{10077696}+\frac{155520}{10077696}+\frac{129600}{10077696}+\frac{174960}{10077696}+\frac{291600}{10077696}+\frac{121500}{10077696}$$
$$+\frac{12960}{10077696}+\frac{43200}{10077696}+\frac{48600}{10077696}+\frac{3600}{10077696}=\frac{1028196}{1007769}\approx 10.2\%$$

If you play the game of Yahtzee with only 4 dice the probability of getting a Yahtzee in a game is 10.2%. This means that a player approximately scores a Yahtzee every every tenth time ($\frac{100}{10.2}\approx 9.8$). The probability of scoring a Yahtzee with 4 dice is again less than the probability of scoring a Yahtzee with 3 dice (21.8%) and 2 dice (42.1%). The probability of socring a Yahtzee with 4 dice is actually 11.6 % less than the probability of scoring a Yahtzee with 3 dice and 31.9% less than the probability of scoring a Yahtzee with 2 dice.

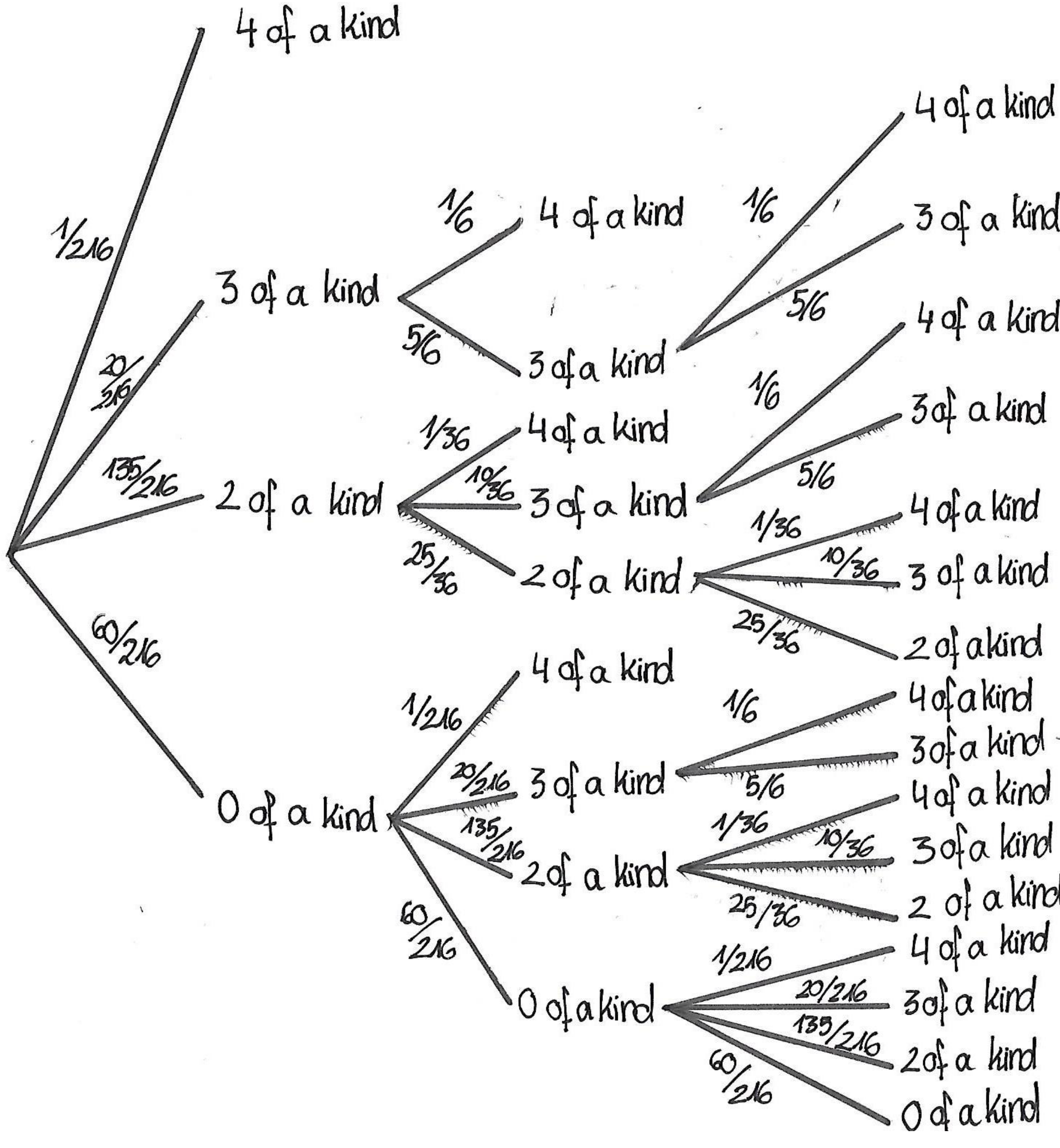


Fig. 7

2.4 Probability of getting a Yahtzee with 5 dice

The game of Yahtzee is normally played with 5 dice. In Fig. 10 (p. 15) you can see the probability tree diagram of all the possible combinations that you can score. The number of possible ways in which a player can score a Yahtzee is even higher than previously as the number of dice increased. In the first roll a player can score a Yahtzee (in the tree diagram: 5 of a kind), a 4 of a kind, a 3 of a kind, a 2 of a kind and a 0 of a kind.

The equation of scoring a Yahtzee in the first roll is:

$$6 \times \left(\frac{1}{6}\right)^5 = \left(\frac{1}{6}\right)^4 = \frac{1}{1296} \approx 0.08\%$$

The probability of scoring a 4 of a kind in the first roll has to be:

$$6 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times 5 = \frac{25}{1296} \approx 1.93\%$$

The probability has to be multiply it by 5 as there exist 5 different combinations in which the dice could be arranged (AAAAB, AAABA, AABAA, ABAAA, BAAAA). In the second roll the player could score a Yahtzee or still have a 4 of a kind. As only one die is left the chance of throwing the right number is 1 to 5.

The probability of scoring a 3 of a kind in the first roll was really hard to find and I couldn't manage to find an equation for it but by again using an Excel spreadsheet of all possible combinations I could simply count all the possible combinations for scoring a 3 of a kind. In Fig. 8 you can find a little section of this Excel spreadsheet.

Scoring Possibilities when playing with 5 Dice					**Numbers the Dice could possibly show**						**Of A Kind**
					6	**5**	**4**	**3**	**2**	**1**	
1	1	1	1	1	0	0	0	0	0	5	5
1	1	1	1	2	0	0	0	0	1	4	4
1	1	1	1	3	0	0	0	1	0	4	4
1	1	1	1	4	0	0	1	0	0	4	4
1	1	1	1	5	0	1	0	0	0	4	4
1	1	1	1	6	1	0	0	0	0	4	4
1	1	1	2	1	0	0	0	0	1	4	4
1	1	1	2	2	0	0	0	0	2	3	3
1	1	1	2	3	0	0	0	1	1	3	3
1	1	1	2	4	0	0	1	0	1	3	3
1	1	1	2	5	0	1	0	0	1	3	3
1	1	1	2	6	1	0	0	0	1	3	3

Fig. 8

There are 7776 possible dice combinations you could get when playing with 5 dice.

Out of these 7776 possible combinations, 1500 combinations were scoring a 3 of a kind which is shown in the table in Fig. 9.

Scoring Possibilities 5 dice	
Of A Kind	**Number of Possible Combinations**
5	6
4	150
3	1500
2	5400
1	720

Fig. 9

So scoring a 3 of a kind when playing with 5 dice must be:

$$\frac{1500}{7776} = \frac{250}{1296} \approx 19.29\%$$

As only 2 dice are left for the second and third roll the probabilities of the tree diagram in Fig. 10 (p. 15) could be used again as they are the same. When the player only used 4 dice before and scored a 2 of a kind in the first roll, 2 dice were left as well. So the following probabilities for the second and third roll are the same in both cases: when a player used 4 dice and scored a 2 of a kind in his first roll or when a player used 5 dice and scored 3 of a kind in his first roll.

Again the probability of getting a 2 of a kind in the first roll when using 5 dice to play the game was really hard to find and I wasn't able to find an equation but by using the same Excel spreadsheet as before I was able to count the number for the possible combinations showing 2 of a kind. There were 5400 possible combinations showing a 2 of a kind as shown in the table in Fig. 9.

So the probability for getting a 2 of a kind in your first roll when playing with 5 dice must be:

$$\frac{5400}{7776} = \frac{900}{1296} \approx 69.44\%$$

After rolling a 2 of a kind in the first roll a player has the chance to roll a Yahtzee, a 4 of a kind, a 3 of a kind or still have a 2 of a kind after the second roll. The probability of getting a 4 of a kind in your second roll has to be $\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times 3 = \frac{15}{216}$. Again we have to be careful here and include the possible combinations of which there are 3 (AAB, ABA, BAA).

The equation to calculate that all dice show a different number would be:

$$6 \times \frac{1}{6} \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} = \frac{120}{1296} \approx 9.26\%$$

Even though the player would probably take a large or a small straight if he didn't score these yet I decided to calculate the probability of scoring a Yahtzee under these circumstances. The probabilities for scoring a Yahtzee, a 4 of a kind, a 3 of a kind, a 2 of a kind or again a 0 of a kind are the same as in the first roll.

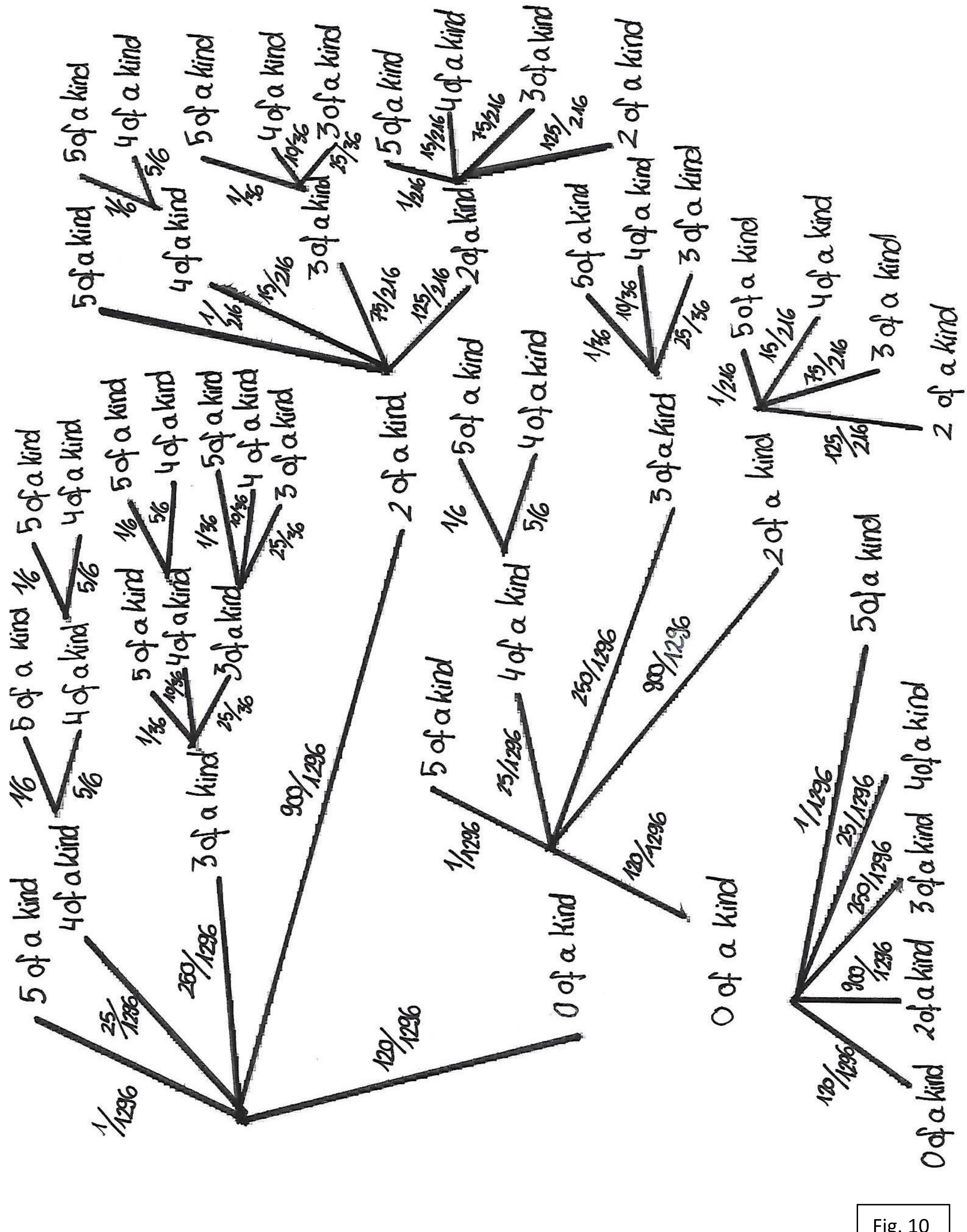

5 of a kind
4 of a kind
3 of a kind
2 of a kind
0 of a kind
1/1296
25/1296
120/1296
900/1296
1/6
5/6
1/36
10/36
25/36
1/216
15/216
75/216
125/216

Fig. 10

There are 15 possible way of scoring a Yahtzee in 3 rolls when the player plays the game with 5 dice. To calculate the overall probability of scoring a Yahtzee with 5 dice I need to add all the probabilities of each single possible way together.

$$\frac{1}{6^4}+\frac{25}{6^5}+\frac{125}{6^6}+\frac{250}{6^6}+\frac{2500}{6^7}+\frac{6250}{6^8}+\frac{900}{6^7}+\frac{13500}{6^8}+\frac{67500}{6^9}+\frac{112500}{6^{10}}+\frac{120}{6^8}+\frac{3000}{6^9}+\frac{30000}{6^{10}}+\frac{108000}{6^{11}}+\frac{120}{6^{12}}$$
$$=\frac{1}{6^4}+\frac{25}{6^5}+\frac{375}{6^6}+\frac{3400}{6^7}+\frac{19870}{6^8}+\frac{70500}{6^9}+\frac{142500}{6^{10}}+\frac{108000}{6^{11}}+\frac{120}{6^{12}}$$
$$=\frac{1679616}{2176782336}+\frac{6998400}{2176782336}+\frac{17496000}{2176782336}+\frac{26438400}{2176782336}+\frac{25751520}{2176782336}+\frac{15228000}{2176782336}+\frac{5130000}{2176782336}+\frac{648000}{2176782336}+\frac{120}{2176782336}=\frac{98786856}{2176782336}$$
$$\approx 4.54\%$$

So if you play the game of Yahtzee as usual with 5 dice the probability of getting a Yahtzee is 4.54%. This means that approximately every 23rd time a player scores a Yahtzee ($\frac{100}{4.54} \approx 22.04$). The probability of scoring Yahtzee when playing with 5 dice is less than the probability of scoring a Yahtzee with 4 dice (10.2%), 3 dice (21.8%) and 2 dice (42.1%).

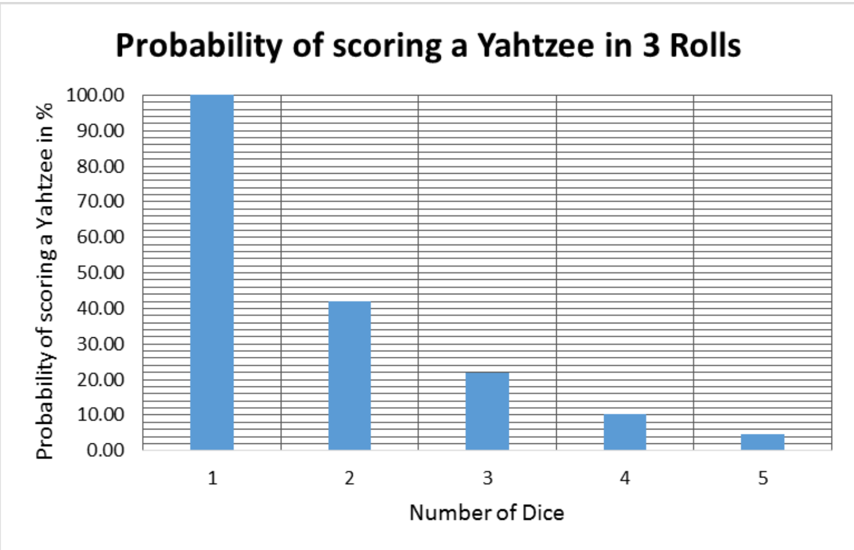
3.0 Conclusion

My aim for this investigation was to calculate the probability of scoring a Yahtzee in 3 rolls when playing with 5 dice. In the beginning I thought that it wouldn't be this hard and started my calculations. But as they turned out they were wrong because I didn't respected the different combinations that were possible in the order of the dice. But after several new ways of calculating the probabilities and varying the number of dice I got answers that seemed right to me. Finding equations for calculating the probabilities of scoring a Yahtzee when playing with 2 and 3 dice was already challenging as I didn't respected the number of possible combinations in the order of the dice but I managed to take these into account which therefore led me to the right equation of calculating the probabilities. On the other hand calculating the probability of scoring a Yahtzee when playing with 4 and 5 dice was even more challenging and it turned out that some probabilities I wanted to calculate were too hard and I chose to use an Excel spreadsheet to find the number of possible combinations. Through the use of this spreadsheet I didn't have to find an equation for calculating 2 of a kind when using 4 dice and 3 and 2 of a kind when using 5 dice. On average, according to my calculated probabilities, a player should score a Yahtzee every 23^{rd} time when playing with 5 dice, every 10^{th} time when playing with 4 dice, every 5^{th} time when playing with 3 dice and every 3^{rd} time when playing with 2 dice as shown in the table in Fig. 11. Logically the number of tries in which one Yahtzee occurs is increasing when the players increase the number of dice they are playing with. Also when I played the game with my family it was recognizable that not everyone scored a Yahtzee in 3 rolls when playing the game with 5 dice, which underlines the fact that a Yahtzee can't be scored in every game. Also you don't always set out to get a Yahtzee on each roll as you might think the probability to score a different combination that you still need in the game, is higher than scoring a Yahtzee. For example you could try to get a Yahtzee but get only a 4 of kind or any other scoring combinations and go for this scoring combinations as you might still need it in the game.

Table to show all Probabilities of scoring a Yahtzee in 3 Rolls

Number of Dice	Probability of Scoring a Yahtzee	Number of Times in which a Player should score a Yahtzee
1	100%	Every time
2	42.1%	Every 3^{rd} time
3	21.8%	Every 5^{th} time
4	10.2%	Every 10^{th} time
5	4.54%	Every 23^{rd} time

Fig. 11



In the graph in Fig. 12 you can see how the probabilities of scoring a Yahtzee vary when using a different number of dice to play the game.

Fig. 12

4.0 Bibliography

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