

# Exploring Tides Times in a Surf Resort

## Introduction

In this assignment I will be investigating the mathematics hidden in the tidal movement throughout a month. My data is based on the tide calendar of Biarritz, a beach resort on the Atlantic coast in France for the month of January 2016<sup>1</sup>. I chose this topic because I go on holiday to this resort reputed for surfing every summer with my family, and my dad always insist on checking the tides before going. Therefore, I was wondering whether it is possible for me to predict at which time the next high tide will be.

## Aim

This exploration aims to investigate the mathematical patterns present within the time of tides in a beach resort as well as in the coefficient of these tides. I will accomplish this aim using a range of mathematical skills learnt in the IB SL Mathematics course.

## Exploration

Figure 1 shows the table that has all the data on which I will base my investigation. Before doing so I will transform every time given in minutes by multiplying the number of hours by 60 and adding it to the number of minutes. The first column shows the time of the first high tide in the morning, the third column shows the second high tide of the day which arrives in the afternoon. The low tides are shown in the fifth and sixth columns. The second and fourth columns show the coefficient of the tides, which the difference in height of consecutive high and low

the tides between two tides.

JANVIER 2016						
Heure Légale						
①	Pleines Mers			Basses Mers		
	Matin	Coef	Soir	Coef	Matin	Soir
	h min		h min		h min	h min
1 V	09 00	052	21 36	047	02 40	15 18
2 S	09 57	043	22 41	040	03 33	16 17
3 D	11 06	037	23 49	037	04 36	17 26
4 L	--	--	12 17	038	05 46	18 34
5 M	00 51	040	13 20	043	06 53	19 29
6 M	01 44	048	14 13	053	07 49	20 15
7 J	02 31	058	14 57	063	08 35	20 56
8 V	03 13	068	15 38	074	09 18	21 34
9 S	03 54	078	16 17	083	09 58	22 13
10 D	04 34	087	16 56	090	10 38	22 52
11 L	05 15	093	17 36	095	11 18	23 33
12 M	05 56	095	18 18	096	--	12 00
13 M	06 40	095	19 02	093	00 15	12 43
14 J	07 26	091	19 49	087	00 59	13 28
15 V	08 16	083	20 42	078	01 47	14 17
16 S	09 12	074	21 43	068	02 39	15 11
17 D	10 16	064	22 54	060	03 40	16 14
18 L	11 28	057	--	--	04 51	17 28
19 M	00 09	056	12 42	056	06 09	18 44
20 M	01 21	059	13 53	062	07 22	19 50
21 J	02 23	067	14 53	072	08 23	20 45
22 V	03 16	076	15 43	081	09 16	21 33
23 S	04 02	085	16 26	088	10 03	22 16
24 D	04 43	090	17 05	092	10 45	22 57
25 L	05 21	092	17 41	092	11 25	23 35
26 M	05 56	091	18 15	089	--	12 03
27 M	06 29	087	18 48	084	00 11	12 39
28 J	07 03	080	19 22	076	00 46	13 13
29 V	07 38	071	19 59	067	01 21	13 50
30 S	08 15	061	20 41	056	01 59	14 29

<sup>31</sup>Figure 1 Raw data

<sup>1</sup> Biarritz Tourisme, 2016. *Calendrier des Marées 2016*, Biarritz: Mairie de Biarritz.

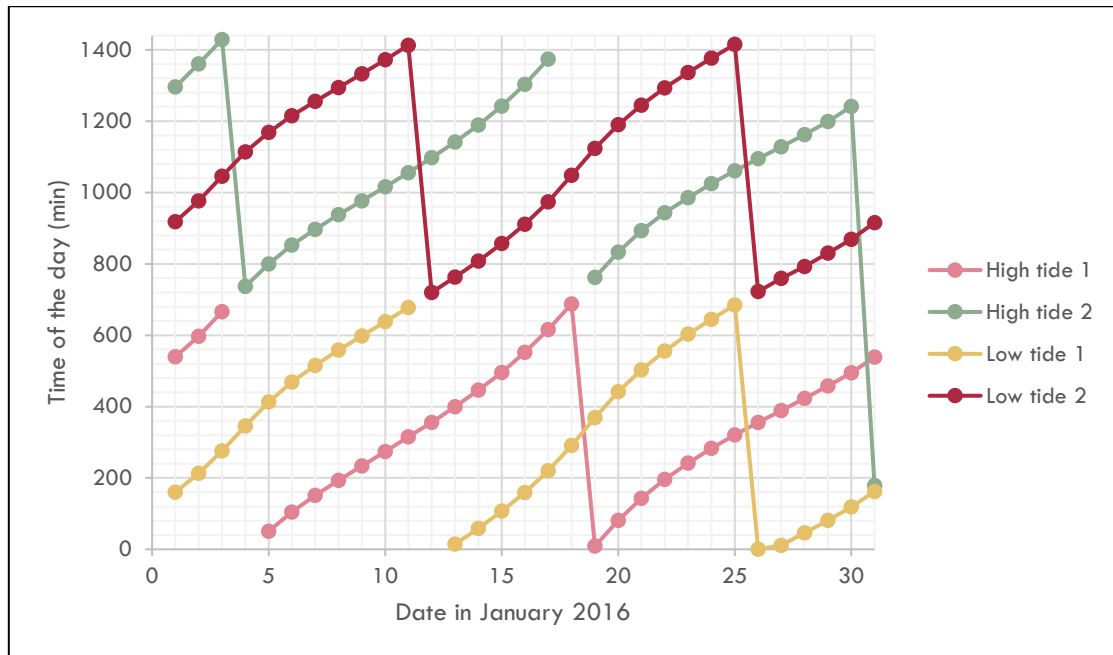


Figure 2 Initial Data

Figure 2 shows the time of tides converted in minutes each day of the month of January 2016. There are 2 high tides and 2 low tides a day which happen later every day until it goes onto the next day. Each tide is shown in a different color in order to differentiate them for each day. The y-axis of this graph only goes up to 1440 because it is the number of minutes in a day (60minx24hours). For example, on the 5<sup>th</sup> of January there are two high tides, the first at 51 minutes or 00:51 and the second at 800 minutes or 13:20. Then, on the 6<sup>th</sup> of January the first high tide is at 104 minutes or 1:44 and the second is at 853 minutes or 14:13. Therefore, the high tide is later by 53 minutes and so is the second high tide.

Looking at this graph I first thought that each tide was made of several linear functions ( $y=ax+b$ ) with the same gradient which means that each day the high and/or low tides are later each day by the same amount of time as it was shown by looking at the 5<sup>th</sup> and 6<sup>th</sup> of July. Consequently, I found the equations of the regression lines for the 3 sections of the second high tide using Excel. In these functions,  $f(x)$  represents the time of the tide in minutes and  $x$  the day on which it happens.

Section 1 stands for  $1 \leq x \leq 3$ , Section 2 stands for  $4 \leq x < 18$ , there is no data for the 18<sup>th</sup> and Section 3 stands for  $19 \leq x \leq 30$ . The 31<sup>st</sup> is not in the continuity of Section 3 and cannot form a function on its own. Therefore I will not take it into account.

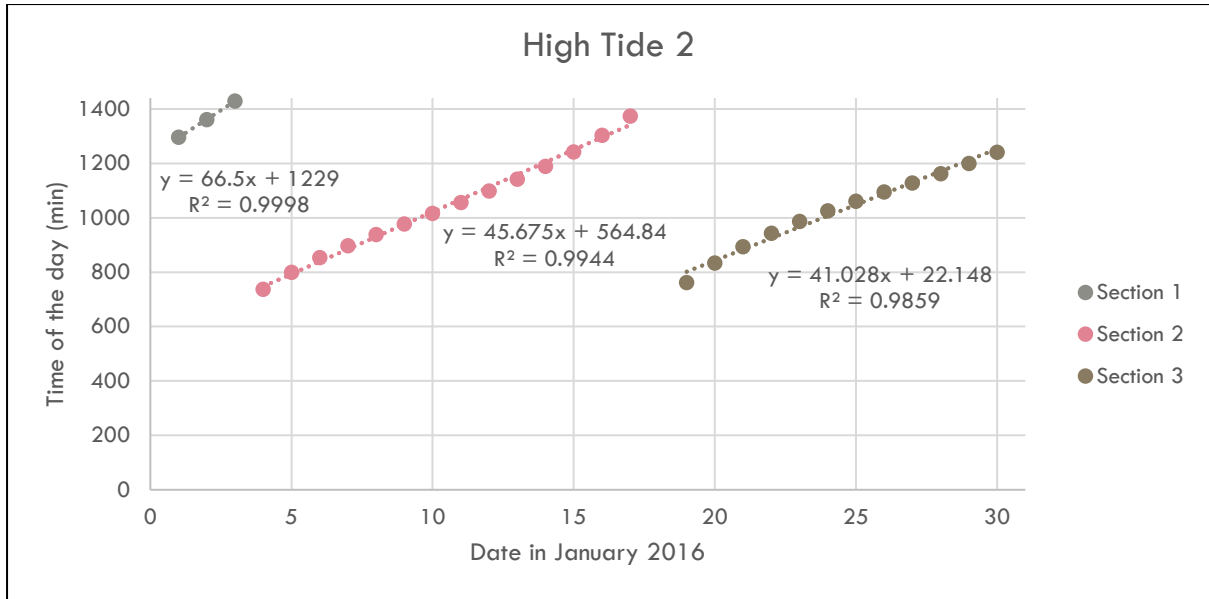


Figure 3 Linear regression lines for High tide 2

This can be written as a piecewise function  $x = \text{date}$  and  $f(x) = \text{time of the day}$ .

$$f(x) = \begin{cases} 66.5x + 1229; & 1 \leq x \leq 3 \\ 45.7x + 565; & 4 \leq x \leq 17 \\ 41.0x + 22.1; & 19 \leq x \leq 30 \end{cases}$$

The  $R^2$  value is a measure of the correlation between two variables  $X$  and  $Y$ , giving a value between +1 and -1 inclusive. It is used here as a measure of the strength of linear dependence between two variables<sup>2</sup>. It is applied here to all the  $X$  and  $Y$  values of each section.

From the  $R^2$  values on the diagram (Figure 3), we see that the correlation between the regression lines and the sections of the High Tide 2 function is strong as it is above 0.75<sup>3</sup>. They are automatically calculated by Excel but can be found using the formula<sup>4</sup>:

$$R^2 = \frac{S_{xy}}{S_x S_y}$$

However, despite this it occurred me that the position of the points in each section looks similar to the basic arcsine function because of their wavy shape and their gradient (see Figure 4). This observation leads me to think that an arcsine function could be a better fit for these points. Consequently, I will investigate how an arcsine function could fit with each section.

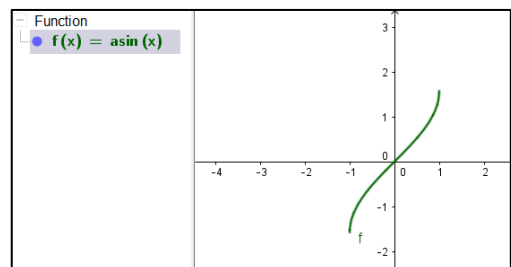


Figure 4 Basic arcsine function

<sup>2</sup> Buchanan, L. et al., 2012. In: *Mathematics Standard Level*. Oxford: Oxford University Press, p. 349.

<sup>3</sup> Buchanan, L. et al., 2012. In: *Mathematics Standard Level*. Oxford: Oxford University Press, p. 350.

<sup>4</sup> Ibid.

I entered the points for  $4 \leq x \leq 17$  in the spreadsheet of the Geogebra program creating List1, however I swapped the  $x$ s and the  $y$ s in order to make the points fit with a sine function as Geogebra does not process arcsine functions. Then, I found the best-fitting sine function to the points (see Figure 5). Thus I obtained the following function:

$$f(x) = 9.96 + 7.53 \sin(0.0033x + 2.92)$$

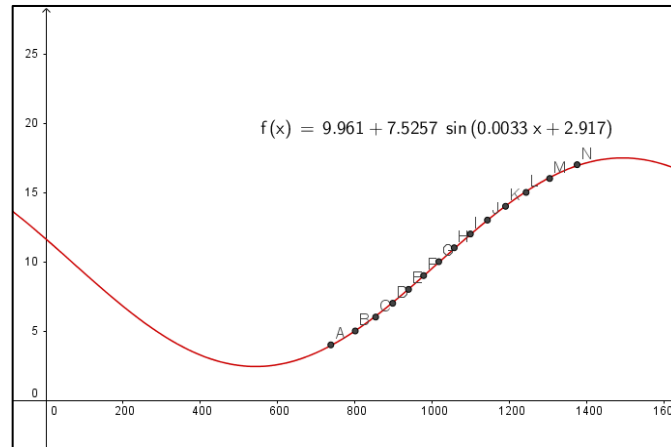


Figure 5 Sine function for Section 2

In this case,  $x$  represents the time in the day and  $f(x)$  shows the date in January 2016.

This function is more closely correlated with the point than the regression line, it is almost a perfect fit as its  $R^2$  value is equal to 0.999. I can now investigate this function by finding its amplitude, period, horizontal and vertical shift:

$$f(x) = 9.96 + 7.53 \sin(0.0033x + 2.92)$$

$$\text{Vertical shift} = 9.96$$

$$\text{Amplitude} = 7.53$$

$$\text{Period} = \frac{2\pi}{0.0033} \approx 1904$$

$$\text{Horizontal shift} = 2.92$$

Forthwith, I can return this function to its original form of arcsine function by swapping  $f(x)$  and  $x$  before isolating  $f(x)$ .

$$x = 9.96 + 7.53 \sin(0.0033 f(x) + 2.92)$$

$$x - 9.96 = 7.53 \sin(0.0033 f(x) + 2.92)$$

$$\frac{x - 9.96}{7.53} = \sin(0.0033 f(x) + 2.92)$$

$$\sin^{-1}\left(\frac{x - 9.96}{7.53}\right) = 0.0033 f(x) + 2.92$$

$$\sin^{-1}\left(\frac{x - 9.96}{7.53}\right) - 2.92 = 0.0033 f(x)$$

$$\left[\sin^{-1}\left(\frac{x - 9.96}{7.53}\right) - 2.92\right] \div 0.0033 = f(x)$$

Hence, the function which finds the time in minutes of the second high tide starting from the date is:

$$f(x) = \left[\sin^{-1}\left(\frac{x - 9.96}{7.53}\right) - 2.92\right] \div 0.0033$$

We can verify the suitability of this function by plotting it with the points (see Figure 6):

From Figure 6 we see that the function found does not fit with our points.

It can be justified by the fact that  $f(x) = \sin x$  is not a one-to-one function, which means that it has more than one  $f(x)$  for a given  $x$  and that it does not have an inverse. Therefore, to find the inverse of  $f(x) = \sin x$  we restrict the domain to  $\pm \frac{\pi}{2}$  which shows the range from the minimum  $f(x)$  value to the maximum.

Moreover, we can also adjust the position of the function by applying a vertical translation in the form of an addition of 1900 (found by trying out) at the end of the function (see Figure 7):

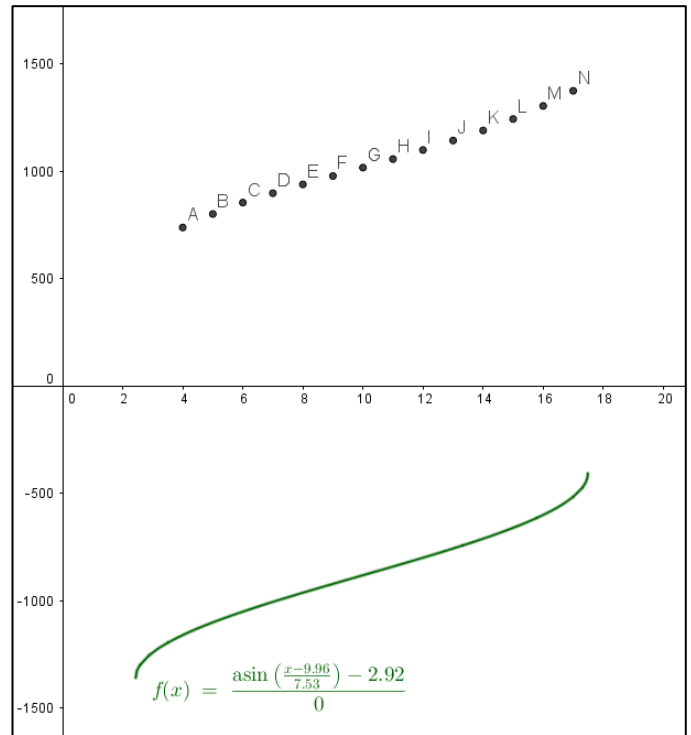


Figure 6 Arcsine function 1

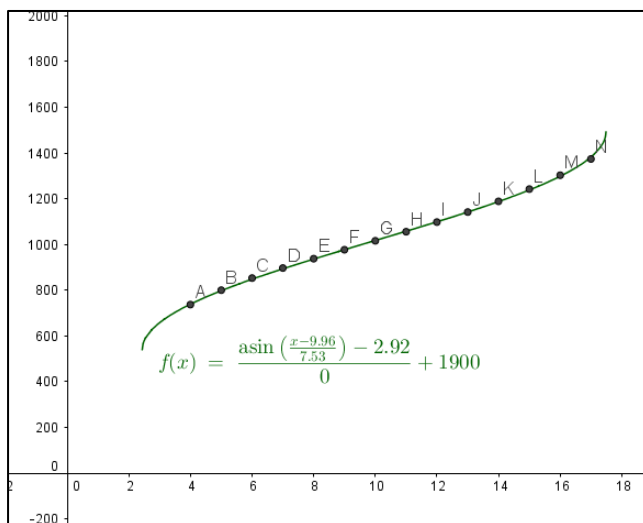


Figure 7 Arcsine function 2

This function is a perfect fit, the  $R^2$  value which is equal to 1 as calculated by Geogebra.

In Figure 2, we see that the function linking a section of the time of High Tide 1 follows the function linking a section of High tide 2 and similarly for the Low tides. I will therefore verify if they fit in with the same function. I will look at High Tide 1 for  $5 \leq x < 18$  and High Tide 2 for  $19 \leq x \leq 30$ . First, I will do the same process as before and find the fitting sine function for each before translating it into arcsine functions that I will compare.

### High Tide 1 for $5 \leq x < 18$ :

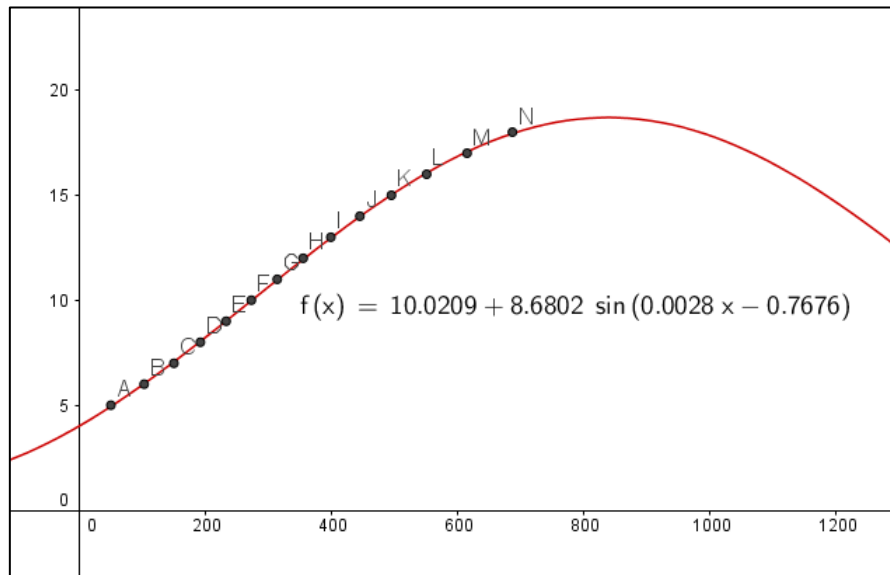


Figure 8 Sine function of High Tide 1 for  $5 < x < 18$

$$g(x) = 10.0 + 8.68 \sin(0.0028x - 0.768)$$

$$x = 10.0 + 8.68 \sin(0.0028f(x) - 0.768)$$

$$g(x) = \left( \sin^{-1} \left( \frac{x - 10.0}{8.68} \right) + 0.768 \right) \div 0.0028$$

The arcsine function linking the time and date of High Tide 1 for  $5 \leq x < 18$  (see Figure 9), where  $x$  represents the time is  $g(x) = \left( \sin^{-1} \left( \frac{x - 10.0}{8.68} \right) + 0.768 \right) \div 0.0028$ .

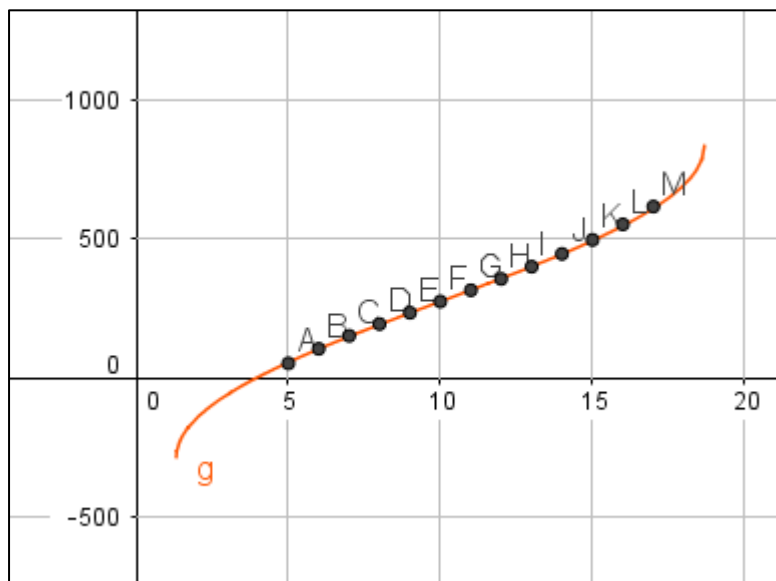


Figure 9 Arcsine function 3

There again it is an almost perfect fit and the  $R^2$  value is equal to 0.999.

## High Tide 2 for $19 \leq x \leq 30$ :

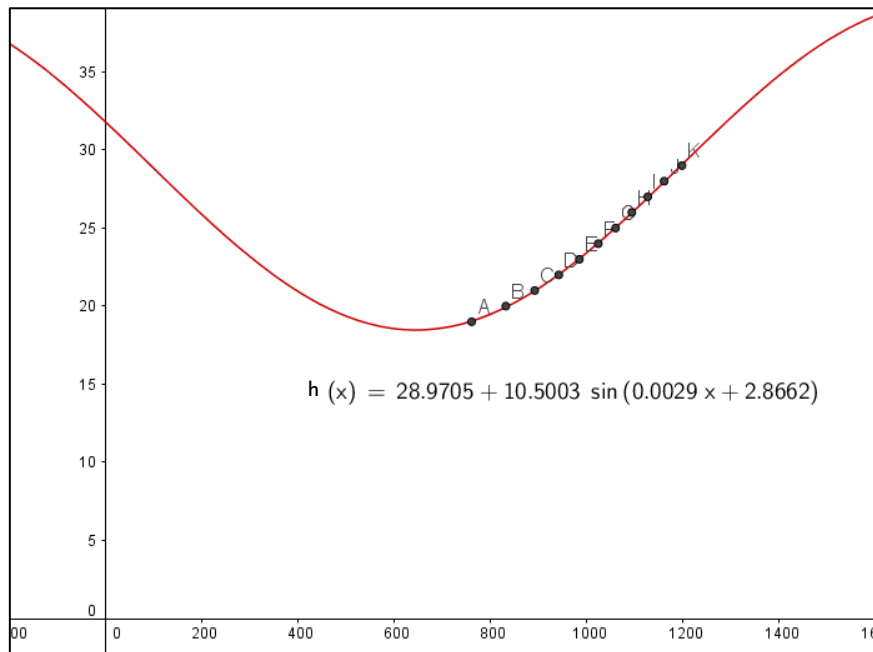


Figure 10 Sine function for High Tide 2 for  $19 < x < 30$

$$h(x) = 28.9705 + 10.5003 \sin(0.0029x + 2.8662)$$

$$x = 28.9705 + 10.5003 \sin(0.0029 h(x) + 2.8662)$$

$$h(x) = \left( \sin^{-1} \left( \frac{x - 28.9705}{10.5003} \right) - 2.8662 \right) \div 0.0029$$

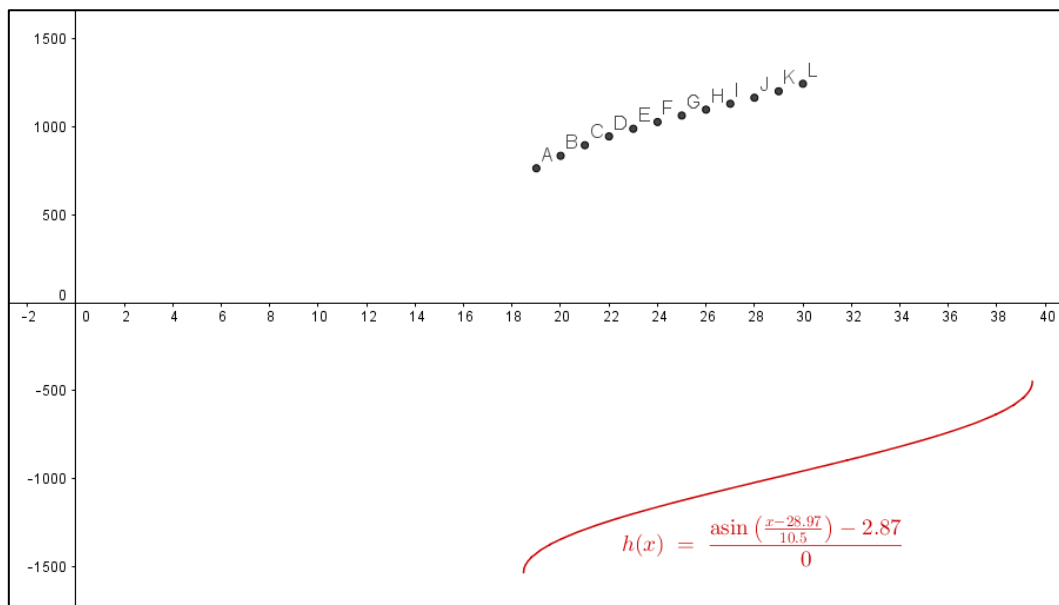


Figure 11 Arcsine function 1 for High Tide 2 for  $19 < x < 30$

The function in Figure 11 does not correspond to the list of points but can be adapted with a vertical shift done by adding 2190 to the expression  $x$ . I found this number by adding in different numbers until I found the one with which the function best fitted to the points.

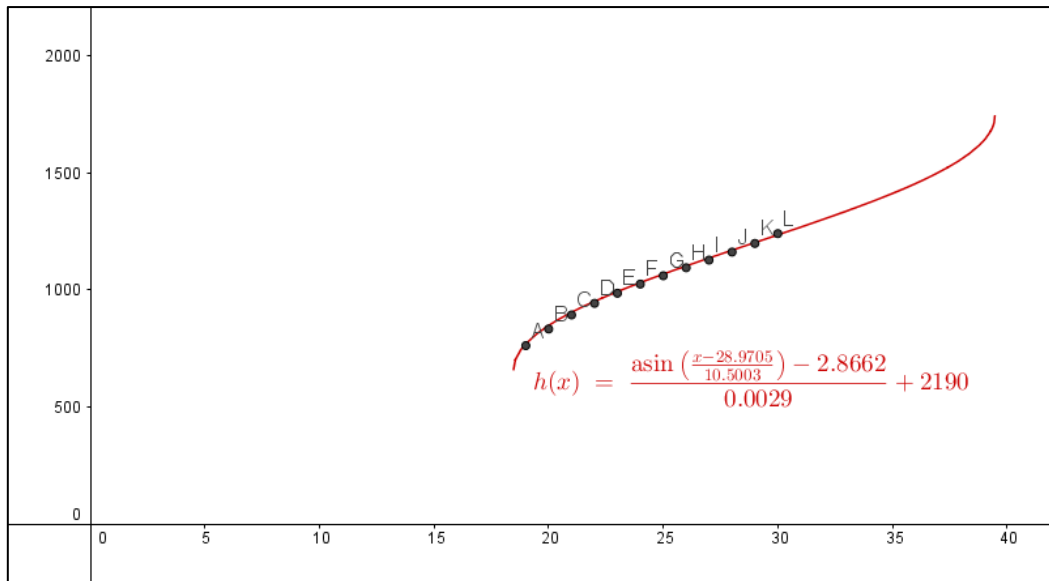


Figure 12 Arcsine function 2 for High Tide 2 for  $19 < x < 30$

We can see on Figure 12 that the arcsine function linking the time and date of High Tide 2 for  $19 \leq x \leq 30$ , where  $x$  represents the time is  $h(x) = (\sin^{-1}(\frac{x-28.9705}{10.5003}) - 2.8662) \div 0.0029 + 2190$ .

Besides, there is a strong correlation between this function and the points as the  $R^2$  value is equal to 0.996.

### Comparison of $g(x)$ and $h(x)$

- $g(x) = (\sin^{-1}(\frac{x-10.0209}{8.6802}) + 0.7676) \div 0.0028$
- $h(x) = (\sin^{-1}(\frac{x-28.9705}{10.5003}) - 2.8662) \div 0.0029 + 2190$

Looking at both these equations we can see that the **period** of their sine forms are very close we see in Figure 13 that the arrows indicating the periods have a very close length. This could mean that the first and the second high tides repeat the same cycle at almost the same frequency from the 5<sup>th</sup> to the 29<sup>th</sup> of January 2016.

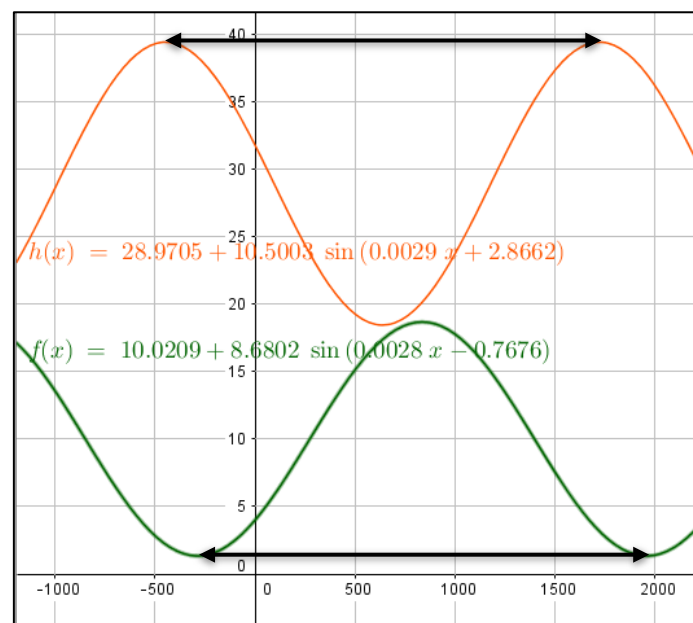


Figure 13 Functions  $g(x)$  and  $h(x)$

Nonetheless, we must remember that it is the arcsine form of these function that corresponds to the original data and in this form these values indicate the **vertical stretch** of the function. Therefore, although both sections do not fit in the same functions as we could have thought looking at Figure 2, they do have their vertical stretch in common which means that both function create undulations of almost the same height (see Figure 14).

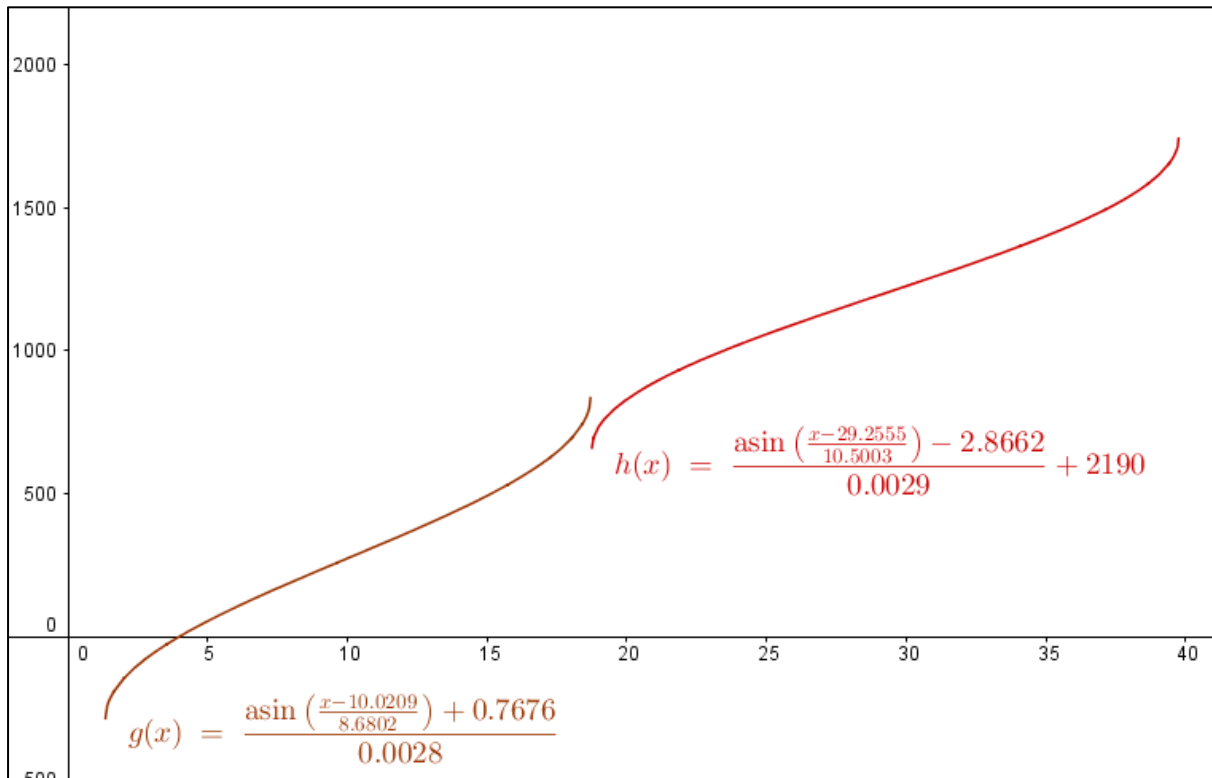


Figure 14 Comparison of h(x) and g(x)

## Coefficients

The tide calendar also includes the coefficient of the tides which represents the difference in height between high and low tide. It is mostly used in France and ranges between 20 and 120<sup>5</sup>. Thus, a big coefficient indicates there are big waves and is useful for swimmers and surfers. I will compare it to the time of the tides to investigate correlation between the two. I will look into the first high and low tide and their coefficient.

<sup>5</sup> Anon., 2011. *Coefficient de marées*. [Online] Available at: <http://marees.free.fr/coefficients.html> [Accessed 01 03 2017].

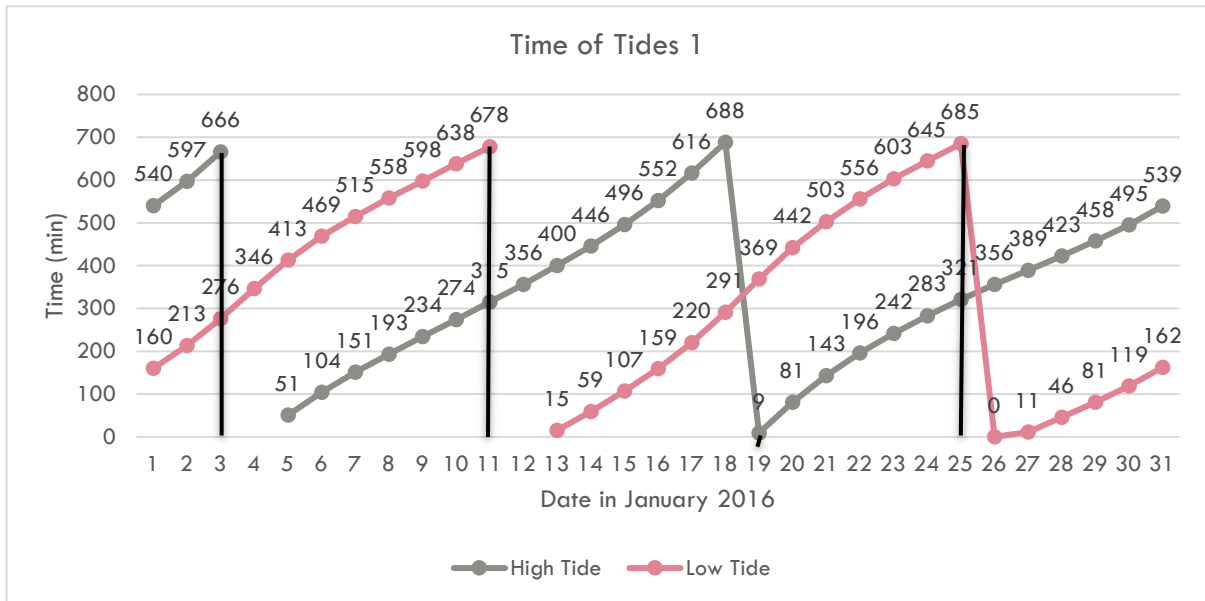


Figure 15 Time of Tides 1

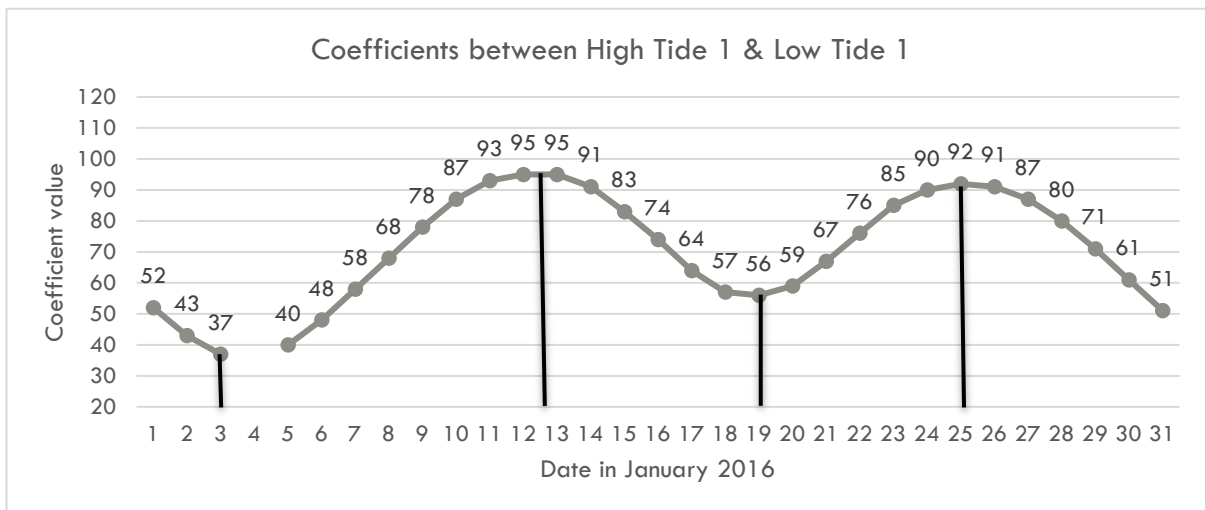


Figure 16 Tide Coefficient 1

When comparing these graphs in Figure 15 and 16, we can see that firstly the function for the coefficient as a sine-like shape, however it isn't because its amplitude is not regular and secondly, the points of inflexion of this graph seem to coincide with the points where the tides arrive earlier rather than later as it is in their usual trend.

Nevertheless, we can find a sine function that will correspond to these points reasonably well using Geogebra.

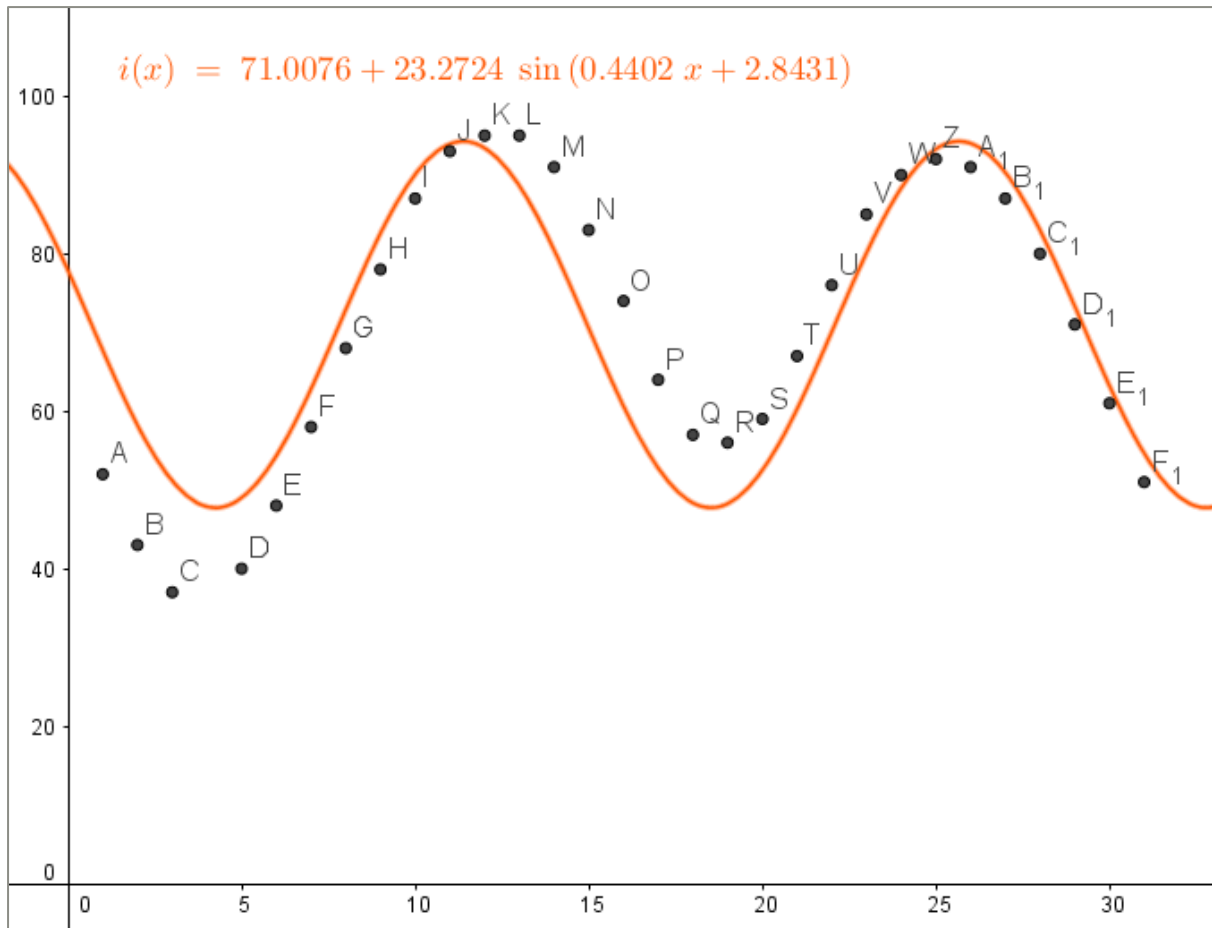


Figure 17 Sine function for the coefficients

We see from figure 17 that the functions  $i(x) = 71.0 + 23.3 \sin(0.440x + 2.84)$  fits to the points quite closely. We can affirm that this function is closely correlated to the tidal coefficients because the  $R^2$  value is equal to 0.804.

Therefore, the coefficients of the tides for the Atlantic Ocean follow the pattern of a sine function.

## Conclusion

Through this investigation, I discovered that the time of tides can be modelled by arcsine functions. Each section of these functions only fit with some sections of the time because the time is fractioned in sections of 24 hours. Moreover, the coefficients of tides fit with a sine function. It is also related to the time of the tides since the date of maximum coefficients almost coincide with the date when the first tide changes trend as it happens earlier rather than later, starting a new section with a new arcsine function associated. Hence, the functions for the time of the tides and their coefficient are related to a circular function. This can probably be explained by the fact that the tides are affected by the moon's gravitational attraction and because the moon orbits on an almost circular path.

## Bibliography

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Anon., 2011. *Coefficient de marées*. [Online]

Available at: <http://marees.free.fr/coefficients.html>

[Accessed 01 03 2017].

Biarritz Tourisme, 2016. *Calendrier des Marées 2016*, Biarritz: Mairie de Biarritz.

Buchanan, L. et al., 2012. In: *Mathematics Standard Level*. Oxford: Oxford University Press, p. 349.

Hohenwarter, M., 2017. *Geogebra*. USA: s.n.

Microsoft, 2013. *Excel*. s.l.:Windows.