Investigating the 'Ideal' and 'Actual' Poppy

Introduction

The centenary of the outbreak of the First World War brought the significance of the remembrance poppy to my attention. The poppy has been used since 1921 to commemorate those who have died in war: behind this symbol of historical significance lies an area of mathematical interest that I intend to explore. The size of the remembrance poppy is similar to that of the actual poppy. Its shape and surface area, however, appear to differ to that of the poppy in nature. This exploration will enable me to apply mathematics in the field of history, which is another source of personal interest to me.

Figure 1: The poppy in nature.

Aim

The aim of this exploration will be to determine the following: whether a graphical model produced is true to the shape of the remembrance and real poppy[1], and the approximate total area of the remembrance poppy leaves. Technology will then be used to assess the accuracy of the surface area value that I have calculated. The shape of the remembrance poppy can be expressed using a model. This same model will then be used to deduce the approximate area of the real poppy, which will then allow me to draw conclusions as to whether or not the remembrance (now to be referred to as the 'ideal') poppy accurately reflects the 'actual' poppy in nature.

Figure 2: The remembrance poppy.

[1]Figure 1, Page 3: Image courtesy of:
http://en.wikipedia.org/wiki/File:Poster_papaver_3a.jpg
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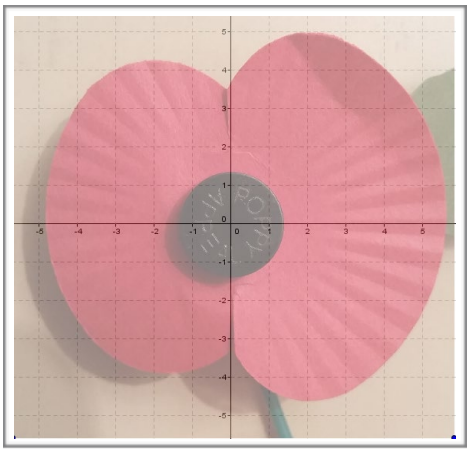
Data Collection

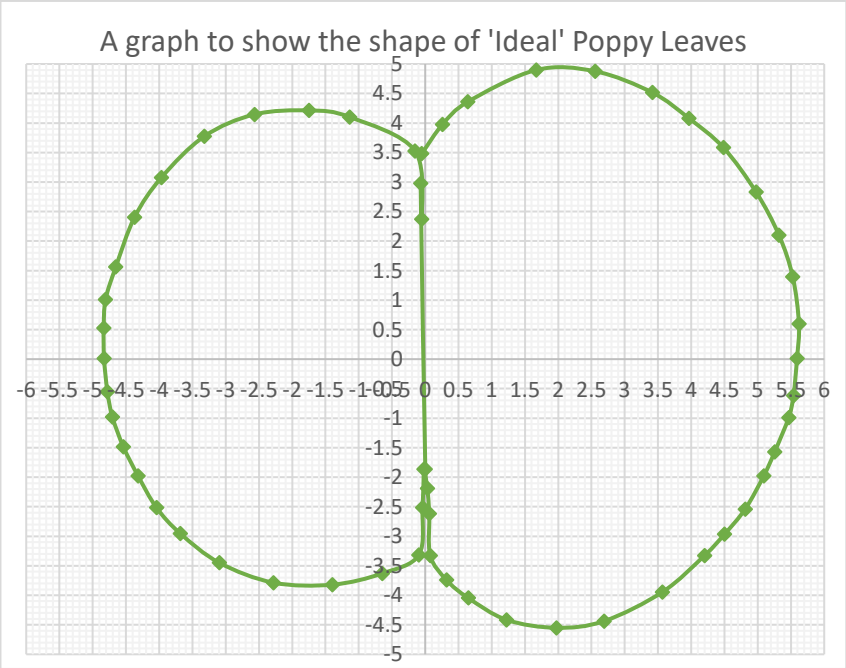
Photographs of both the 'ideal' and 'actual' poppy have been used as the basis of this exploration. These photographs will be used to model both variants of the poppy. In order to model them, I have marked several points along it and used these points to determine the x- and y- coordinates of its shape. The centre of the flowers will act as the origin whilst I model their shapes.

Plotting the Shape of the 'Ideal' Poppy

A photo of the 'ideal' poppy was used to model its shape. Plotted coordinates, as shown in the photos below, acted as the basis of the modelling process. 56 coordinates were plotted in total. These coordinates were then mapped on a computer-based modelling program to ensure that they accurately reflected the 'ideal' poppy shape. The centre of the black circle acted as the origin (0,0). It should be noted that the coordinates are only accurate up to a certain extent, and that errors may have occurred whilst they were plotted. The use of a superimposed axis may also present some scaling issues.



This table shows the X and Y coordinates of the 'ideal' poppy shape, which has then been plotted as a graph.

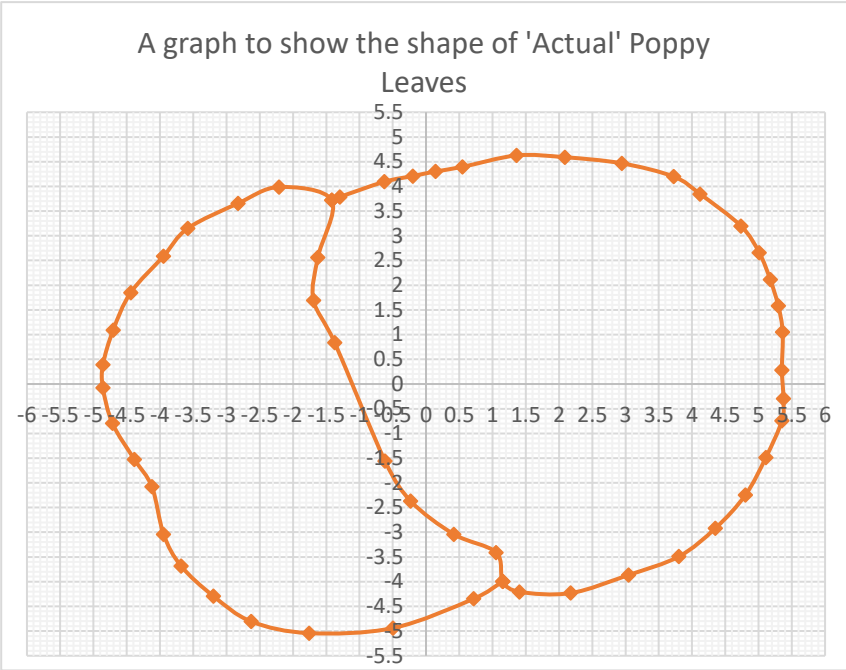


x	*y*
-0.051	3.483
0.261	3.978
0.640	4.360
1.669	4.899
2.555	4.876
3.415	4.517
3.967	4.079
4.484	.584
4.975	2.831
5.319	2.101
5.527	1.393
5.620	0.596
5.591	0.011
5.538	-0.618
5.466	-0.989
5.255	-1.573
5.091	-1.978
4.812	-2.539
4.500	-2.966
4.201	-3.326
3.569	-3.944
2.688	-4.438
1.971	-4.551
1.222	-4.416
0.647	-4.045
0.321	-3.742
0.076	-3.326
0.061	-2.618
0.032	-2.191
0.001	-1.865
-0.069	2.978
-0.053	2.371
-0.153	3.528
-1.135	4.101
-1.748	4.213
-2.567	4.146
-3.321	3.775
-3.965	3.079
-4.371	2.404
-4.653	1.562
-4.807	1.011
-4.835	0.528
-4.830	0.011
-4.779	-0.551
-4.704	-0.978
-4.540	-1.483
-4.318	-1.978
-4.040	-2.517
-3.681	-2.955
-3.096	-3.449
-2.282	-3.787
-1.396	-3.820
-0.644	-3.629
-0.095	-3.315
-0.040	-2.517

Plotting the Shape of the ‘Actual’ Poppy

I then repeated this process to determine the shape of the ‘actual’ poppy, using the photographs attached below. There were 54 coordinates in total. Again, the centre of the plant acted as the origin. The table below shows the X and Y coordinates of the ‘actual’ poppy shape, which has then been plotted as a graph.

x	*y*
-1.418	3.718
-1.297	3.786
-0.626	4.093
-0.198	4.209
0.143	4.302
0.549	4.395
1.363	4.626
2.088	4.590
2.945	4.466
3.725	4.197
4.121	3.846
4.736	3.197
5.011	2.656
5.176	2.113
5.297	1.581
5.363	1.049
5.352	0.282
5.374	-0.296
5.352	-0.740
5.110	-1.488
4.802	-2.247
4.352	-2.918
3.802	-3.491
3.044	-3.866
2.176	-4.231
1.407	-4.206
1.154	-3.998
1.055	-3.410

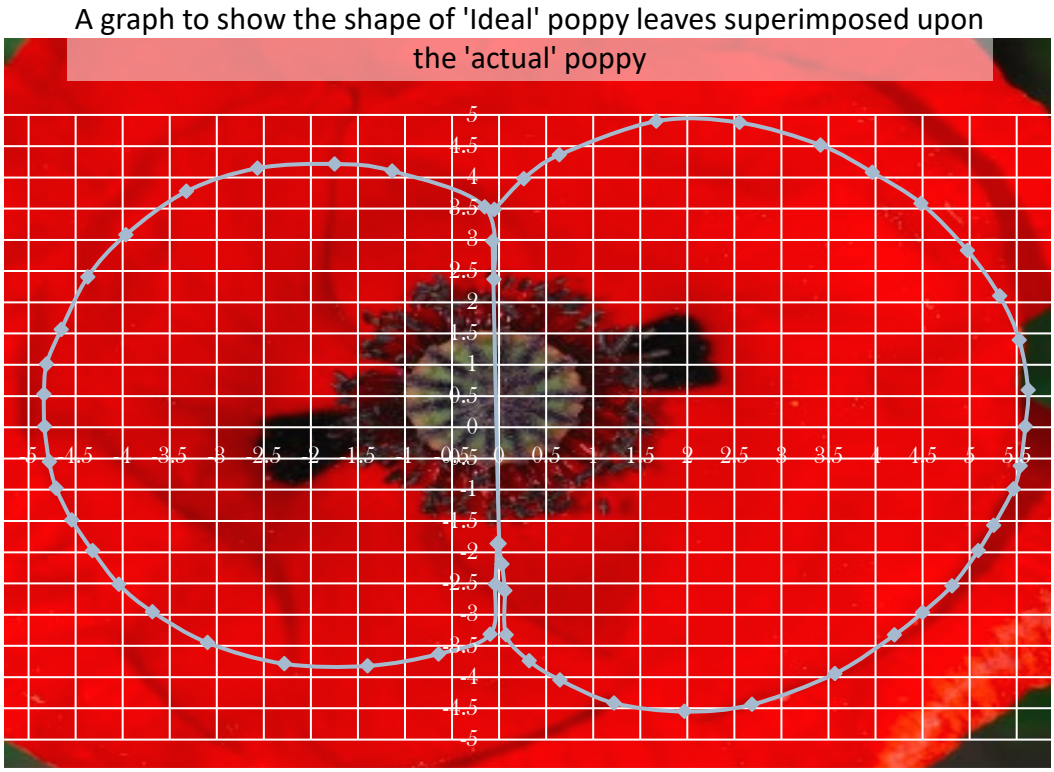


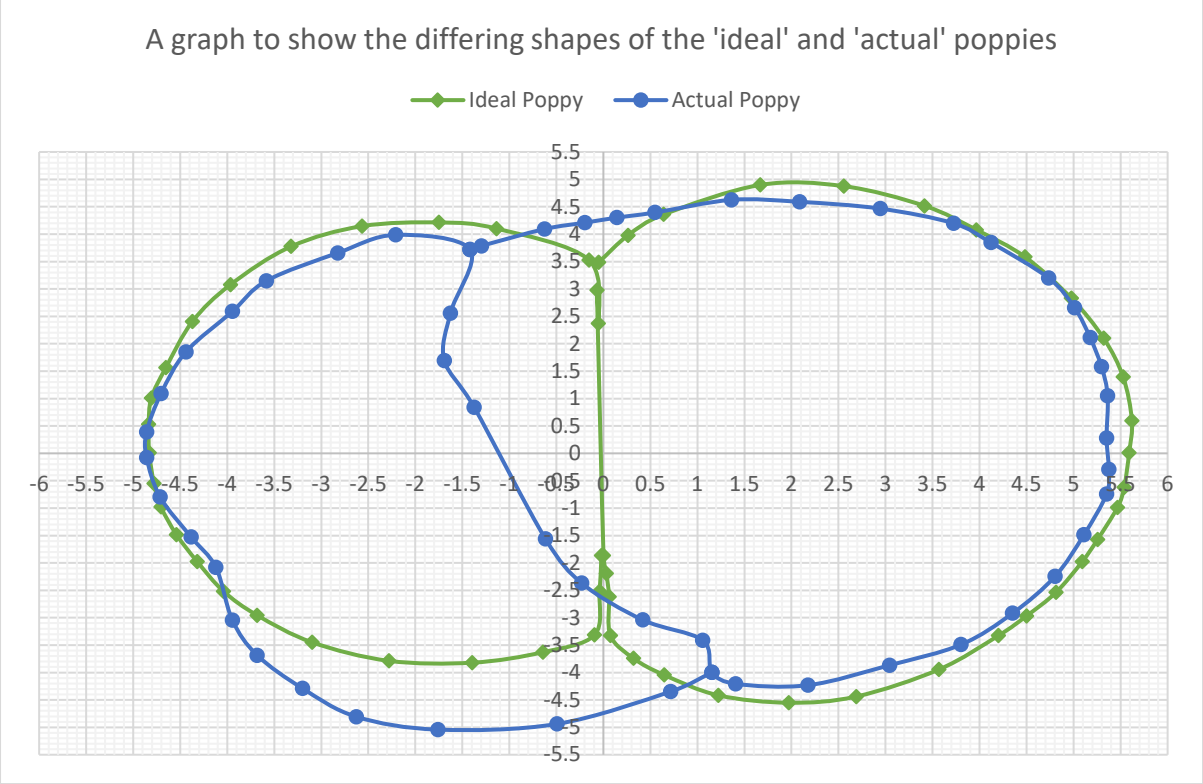
0.418	-3.040
-0.231	-2.369
-0.615	-1.562
-1.374	0.840
-1.692	1.692
-1.626	2.560
-1.418	3.718
-2.209	3.987
-2.824	3.658
-3.582	3.149
-3.945	2.590
-4.440	1.851
-4.703	1.092

-4.857	0.391
-4.857	-0.076
-4.714	-0.797
-4.385	-1.526
-4.121	-2.079
-3.945	-3.044
-3.681	-3.685
-3.198	-4.291
-2.626	-4.807
-1.758	-5.042
-0.495	-4.939
0.714	-4.347
1.154	-3.998

Comparing the Shapes of the ‘Ideal’ and ‘Actual’ Poppies

The qualitative data obtained when plotting the compiled data suggests that the ‘ideal’ poppy does not reflect the ‘actual’ poppy in terms of its general shape. This becomes clear when the graph for the ‘ideal’ poppy is placed on the image of the ‘actual’ poppy.

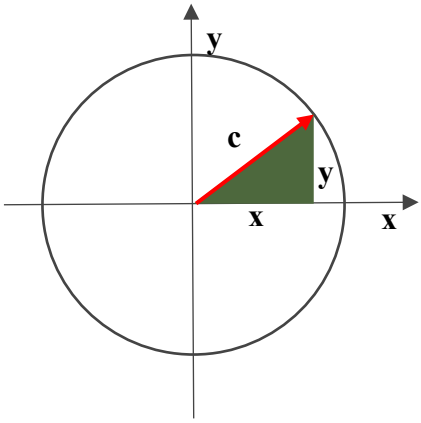




Data Processing

Modelling the 'Ideal' Poppy

Using the plotted coordinates, I was able to determine the general shape of the 'ideal' poppy; it was clear that both leaves resemble a squashed semi-circle, or an ellipse. Given that I know the x- and y- coordinates of the equation, the circle equation,$(x-a)^2+(y-b)^2=r^2$, can be used to help me determine the equation of the leaf shape (where (a, b) gives the centre of the circle and r = radius):

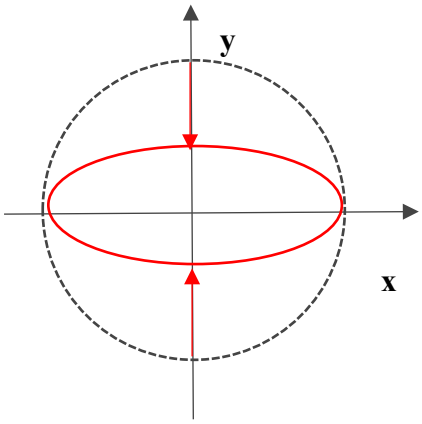


Given that I know the values of 'x' and 'y' (the coordinates), the value of 'c', or the hypotenuse, can be determined through the following equation:

$$\sqrt{x^2+y^2} \quad =\sqrt{c^2}$$

The equation of the circle can therefore be expressed as:

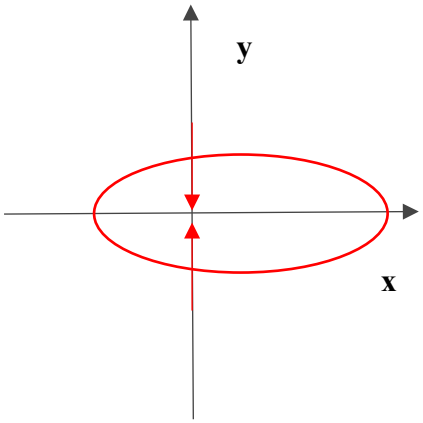
$$y=\pm(c^2-x^2)^{0.5}$$



Upon closer inspection, it is apparent that each 'leaf' of the poppy is a segment of an ellipse. Given that this is the case, it is necessary to compress the circle from its original form. The expression for the diagram can be expressed as:

$$y=\pm a(c^2-x^2)^{0.5}$$

Where 'a' is the factor by which the circle is vertically stretched or compressed.



Given that the ellipse has been shifted to the right along the x-axis, it is again necessary to alter the equation in order to better reflect the actual shape of the 'ideal' right poppy leaf. The equation of the shifted poppy can be expressed as:

$$y=\pm a(c^2-(x-b)^2)^{0.5}$$

Where 'b' is the value by which the ellipse is shifted across the x-axis.

The final equation which I will now manipulate according to the shape of the right leaf of the 'ideal' poppy is:

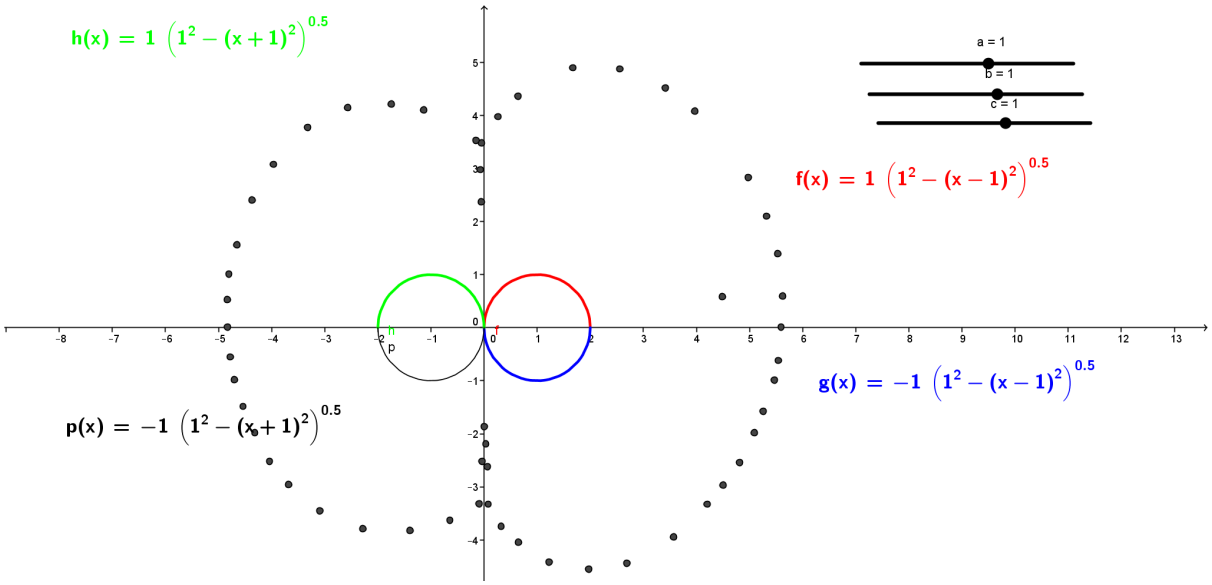
$$y=\pm a(c^2-(x-b)^2)^{0.5}$$

In order to plot the shape of the left leaf, 'negative' values must be plotted. They will be referred to as '**a**'", "**b**'" and "**c**'". By altering these values, I can determine the equation that best fits the actual shape of the 'ideal' leaves. By adding the '**b**' parameter to '**x**' (x+b), the ellipse will shift to the left along the x-axis.

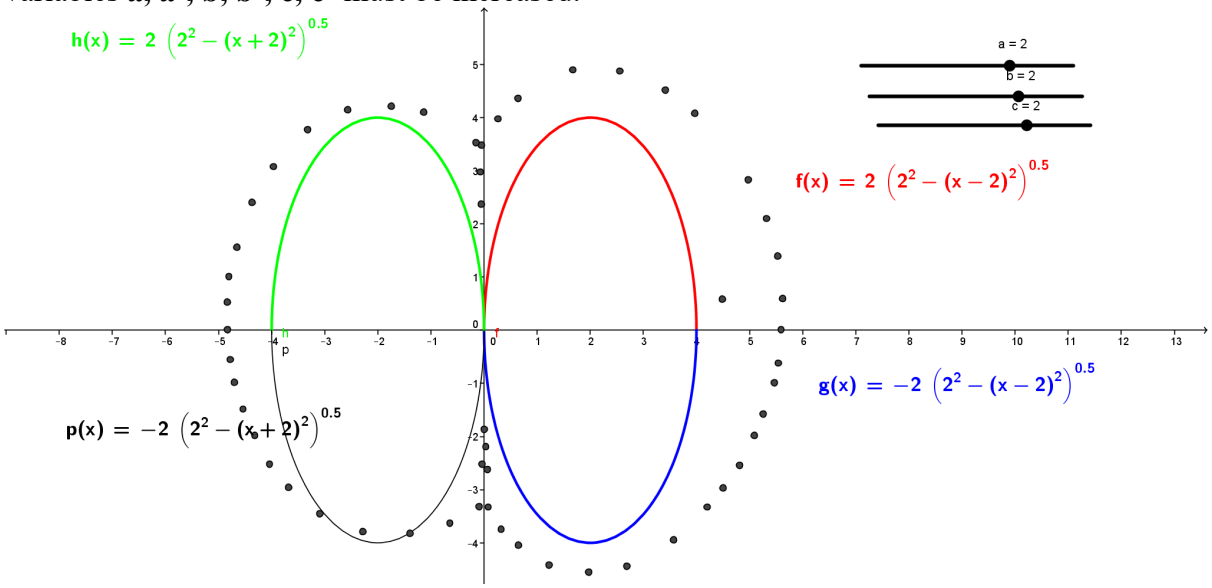
The final equation which I will now manipulate according to the shape of the left leaf of the 'ideal' poppy is therefore:

$$y = \pm a(c^2 - (x + b)^2)^{0.5}$$

Using technology, I can alter the parameters to plot an equation that fits the shape of the 'ideal' poppy.

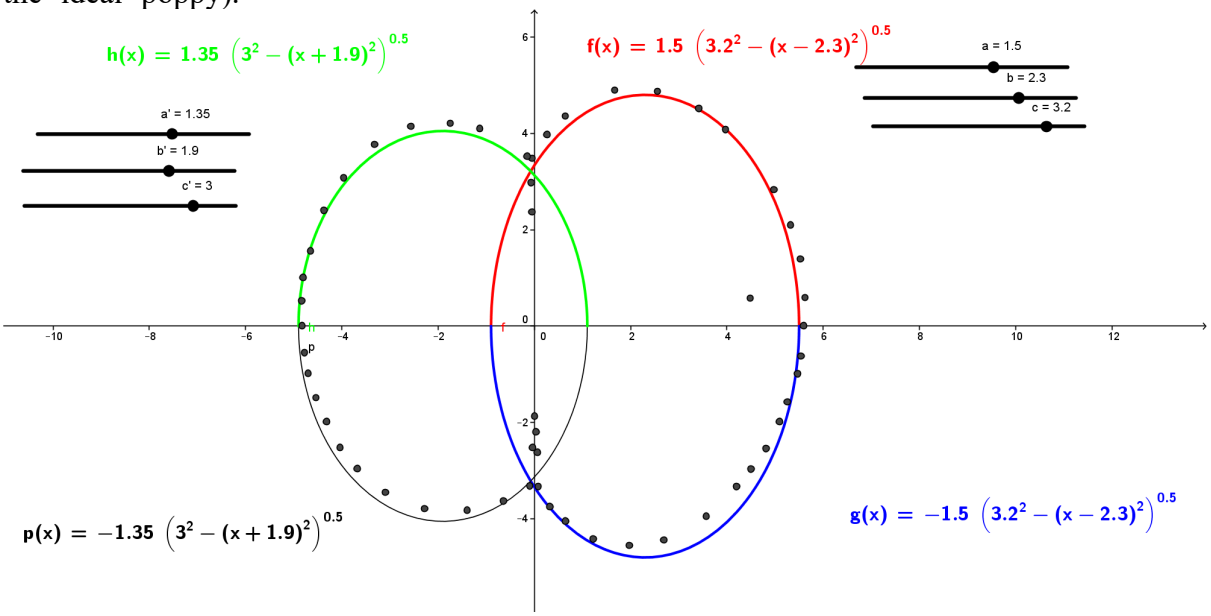


When every parameter (**a, a', b, b', c, c'**) is equal to 1, two small circles are produced, with centres (1,0) and (-1,0). In order to increase the vertical and horizontal stretch of the circles, variables **a, a', b, b', c, c'** must be increased.



When each variable is equal to 2, two larger ellipses are produced. It is clear that variables **c** and **c'** must be increased to increase the horizontal stretch of the ellipses.

After several attempts, I was able to determine the values that best fitted the ellipses (shape of the 'ideal' poppy).



Equation:

Right Leaf: $y = \pm a(c^2 - (x-b)^2)^{0.5}$

Left Leaf: $y = \pm a'(c'^2 - (x+b')^2)^{0.5}$

The values of the parameters were:

a	1.50	a'	1.35
b	2.3	b'	1.9
c	3.2	c'	3

The equation for the right ellipse (right poppy leaf) is therefore:

$$y = \pm(1.50(3.2^2 - (x-2.3)^2)^{0.5})$$

The equation for the left ellipse (left poppy leaf) is therefore:

$$y = \pm(1.35(3^2 - (x+1.9)^2)^{0.5})$$

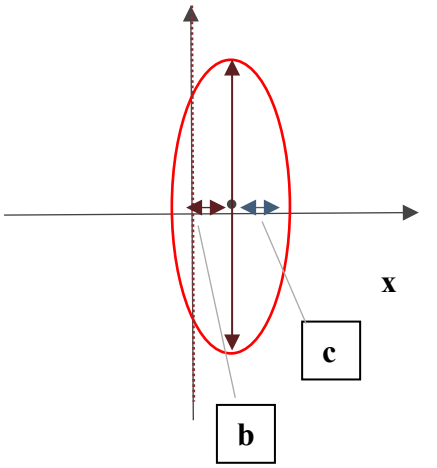
Calculating the Area of the 'Ideal' Poppy

Given that I have determined the approximate equations for both ellipses, it is possible to determine the total area of the 'ideal' poppy. Upon observing the shapes of the ellipses, it is clear that the line $y = 0$ acts as the chord for both equations. If the height of an ellipses is **A**, its width **B**, and the height of the elliptical segment **h**, then the area contained by the ellipse is given by the equation:

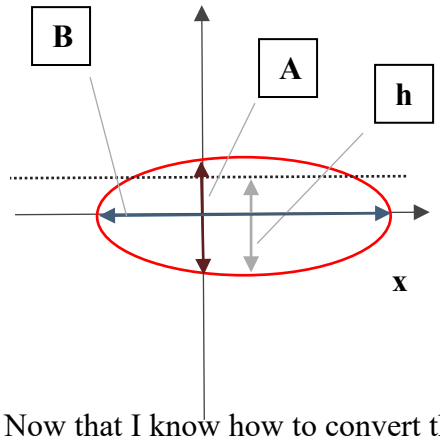
$$Area = \left(\frac{\boldsymbol{AB}}{4}\right)\left[\arccos\left(1 - \frac{2\mathbf{h}}{\mathbf{A}}\right) - \left(1 - \frac{2\mathbf{h}}{\mathbf{A}}\right)\left(\frac{4\mathbf{h}}{\mathbf{A}} - \frac{4\mathbf{h}^2}{\mathbf{A}^2}\right)\right] ([2])$$

It should be said that the values **a** and **b** are not equal to the values **A** and **B**, although their relationship will be explained below. The equation only applies where *arccos* is in radians.

[2] Ellipse Area Equation. Courtesy of:
http://www.had2know.com/academics/ellipse-segment-tank-volume-calculator.html



In this diagram, the value '**c**' represents the radius of the ellipse from its central point. The dotted line represents the chord of the ellipse. The value '**b**' represents the value by which the ellipse has been shifted across the x-axis.



Given that I know the values '**a**', '**b**' and '**c**', it is possible to work out the values '**A**', '**B**' and '**h**'. The value '**A**' is equal to **2c**, as '**c**' is the vertical radius of the ellipse from its central point. '**B**' is equal to **2ac**, as it is dependent on both the radius and the stretch of the ellipse. '**h**' is equal to **(b + c)**, as it is the distance from the bottom of the ellipse to the chord.

Now that I know how to convert the values used to plot the original modelling equation, I can use the formula of the area of a segment to determine the approximate area of each poppy leaf.

$$Area = \left(\frac{\boldsymbol{AB}}{4}\right)\left[\arccos\left(1-\frac{\mathbf{2h}}{\mathbf{A}}\right)-\left(1-\frac{\mathbf{2h}}{\mathbf{A}}\right)\left(\frac{\mathbf{4h}}{\mathbf{A}}-\frac{\mathbf{4h^2}}{\mathbf{A^2}}\right)\right]$$

A=2c
B=2ac
h=(b + c)

<u>Right Leaf</u>
a = 1.5
b = 2.3
c = 3.2

A = 2*3.2 = 6.4
B = 2*1.5*3.2 = 9.6
h = (2.3+3.2) = 5.5

$$\left(1-\frac{2\boldsymbol{h}}{\boldsymbol{A}}\right) = (1-\left(2\cdot\frac{5.5}{6.4}\right) \approx -0.719$$

$$\left(\frac{4\boldsymbol{h}}{\boldsymbol{A}}-\left(\left(\frac{4\boldsymbol{h}}{\boldsymbol{A}}\right)^2\right)\right) = \left(\left(4\cdot\frac{5.5}{6.4}\right)-\left(\left(4\cdot\frac{5.5}{6.4}\right)^2\right)\right) \approx 0.483$$

$$Area\ of\ ellipse\ chord = \left(\frac{\boldsymbol{AB}}{4}\right)\left[\arccos\left(1-\frac{\mathbf{2h}}{\mathbf{A}}\right)-\left(1-\frac{\mathbf{2h}}{\mathbf{A}}\right)\left(\frac{\mathbf{4h}}{\mathbf{A}}-\frac{\mathbf{4h^2}}{\mathbf{A^2}}\right)\right] =$$

$$\left(\frac{6.4\cdot 9.6}{4}\right)[\arccos(-0.719)-(-0.719)\cdot(0.483))) \approx \mathbf{44.1\ units^2.}$$

The area of the right ellipse segment, and therefore the approximate area of the right 'ideal' poppy leaf, is **44.1** units².

Left Leaf
a' = 1.35
b' = 1.9
c' = 3

A=2c'
B=2a'c'
h=(b'+c')

A = 2·3 =6
B = 2·1.35·3 = 8.1
h = (1.9+3) = 4.9

$$\left(1-\frac{2\boldsymbol{h}}{\boldsymbol{A}}\right)=(1-\left(2\cdot\frac{4.9}{6}\right)\approx\ -0.633$$

$$\left(\frac{4\boldsymbol{h}}{\boldsymbol{A}}-\left(\left(\frac{2\boldsymbol{h}}{\boldsymbol{A}}\right)^{2}\right)\right)=\left(\left(4\cdot\frac{4.9}{6}\right)-\left(\left(4\cdot\frac{4.9}{6}\right)^{2}\right)\right)\approx 0.598$$

$$Area\ of\ ellipse\ chord=\left(\frac{\boldsymbol{AB}}{4}\right)\left[\arccos\left(1-\frac{2\mathbf{h}}{\mathbf{A}}\right)-\left(1-\frac{2\mathbf{h}}{\mathbf{A}}\right)\left(\frac{4\mathbf{h}}{\mathbf{A}}-\frac{4\mathbf{h}^{2}}{\mathbf{A}^{2}}\right)\right]\approx$$

$$\left(\frac{6\cdot 8.1}{4}\right)[\arccos(-0.633)-(-0.633)\cdot(0.598)))\approx\boldsymbol{33.4\ units^{2}}$$

The area of the left ellipse segment, and therefore the approximate area of the left 'ideal' poopy leaf, is **33.4** units².

The total area of the poppy, according to my calculations, is **77.5** units².

Using technology, I can determine the accuracy of my calculations. Integration can be used to determine the area of the poppy. It is necessary to integrate each function individually:

Right Leaf
f(x) = 1.5 (3.2²-(x-2.3)²)$^{0.5}$
$\int_0^{5.5}(1.5(3.2)^2-(x-2.3)^2)^{0.5})dx=$ **22.06** units².

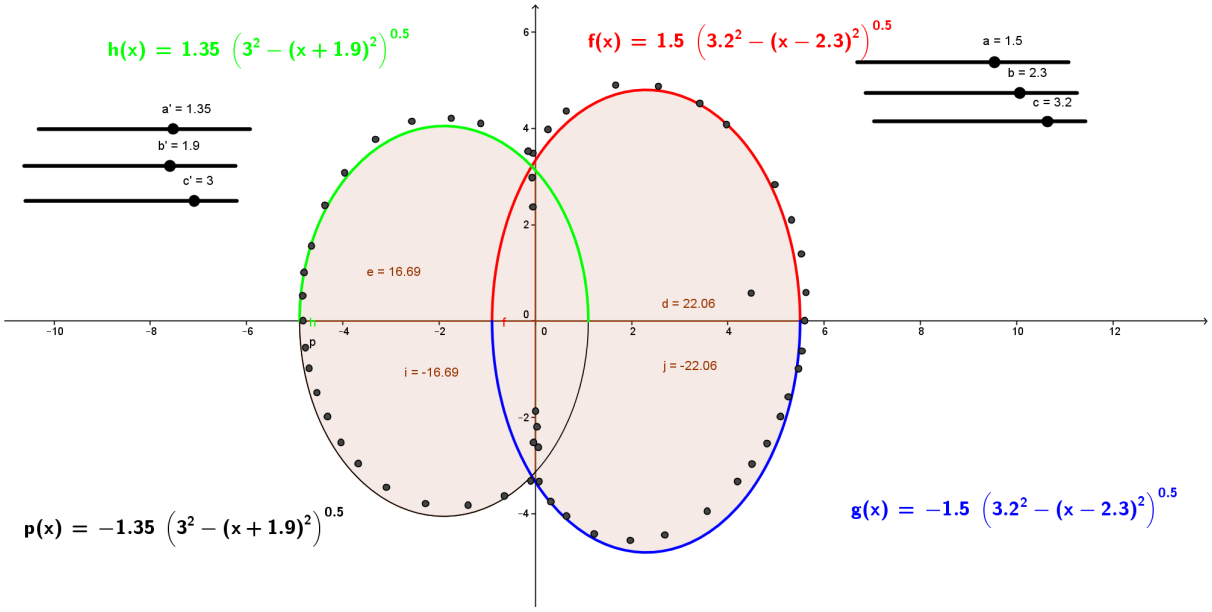
g(x) = -1.5 (3.2²-(x-2.3)²)$^{0.5}$
$\int_0^{5.5}-(1.5(3.2)^2-(x-2.3)^2)^{0.5})dx=$ -22.06 units² (**22.06 units²**).

The area of the right leaf is therefore approximately **44.1 units².**

Left Leaf
h(x) = 1.35 (-3)²-(x+1.9)²)$^{0.5}$
$\int_{-4.9}^{0}(1.35(-3)^2-(x+1.9)^2)^{0.5})dx=$ **16.69 units².**

p(x) = -1.35 (-3)²-(x+1.9)²)$^{0.5}$
$\int_{-4.9}^{0}(-1.35(-3)^2-(x+1.9)^2)^{0.5})dx=$ -16.69 units² (**16.69 units²**).

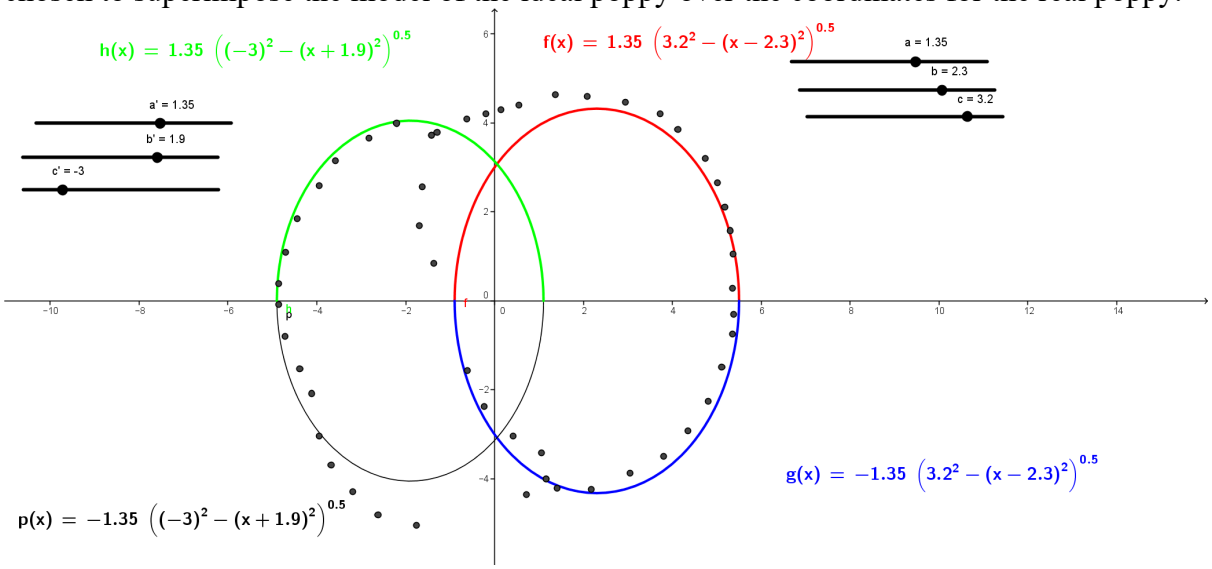
The area of the left leaf is therefore approximately **33.4 units².**



According to technology, the total area of the 'ideal' poppy is approximately equal to **77.5** units², which correlates with the results of my calculations.

Modelling and Calculating the Area of the 'Actual' Poppy

In order to develop a meaningful comparison between the two poppy variants, I need to compare their areas. Given the difficulty in modelling a function to the real poppy, I have chosen to superimpose the model of the ideal poppy over the coordinates for the real poppy.



It is clear that the model used for the ideal poppy does not adequately fit the coordinates for the real poppy. Given its shape, the ellipse model used does not truly reflect its shape. It does, however, enable me to deduce the approximate area of the poppy. Given the constraints of the model, it can only be deduced that the approximate total area of the 'actual' poppy is greater than 77.5 units².

Conclusion

The total area of the remembrance poppy is approximately 77.5 units^2. It should be noted that the central flower (black circle) is excluded from this calculation. Given the complexity of the 'ideal' poppy shape, its curve cannot be expressed as a single equation. It is clear, given the disparity between the shapes of both the remembrance and actual poppy, that the remembrance poppy does not accurately reflect the shape of the real poppy. The model produced is true to the remembrance poppy, but not to that of the poppy in nature. The use of technology has enabled me to assess the accuracy of my calculations, and suggests that they are indeed correct.

The greatest source of limitation within the investigation is that the integration function was not calculated by hand, but instead by a computer which may be susceptible to errors. This exploration could be extended by finding examples of other plants and modelling similar curves to the shape of their leaves. It would also be wise to base the axes unit against an actual unit of measurement (e.g. 0,0 to 1,1 is equal to 1cm^2) in order to obtain a better understanding of the actual area of the plants. Furthermore, multiple functions could be plotted to assemble a model for the area of the 'actual' poppy, allowing me to obtain a more accurate value of its total area (unlike the deduction made using the ellipse model during my investigation). A further point of exploration that could improve the investigation would involve deriving the formula for the area of an ellipse, rather than using an externally sourced formula.

Bibliography

Poppy Image. Courtesy of:
http://en.wikipedia.org/wiki/File:Poster_papaver_3a.jpg
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Ellipse Area Equation. Courtesy of:
http://www.had2know.com/academics/ellipse-segment-tank-volume-calculator.html